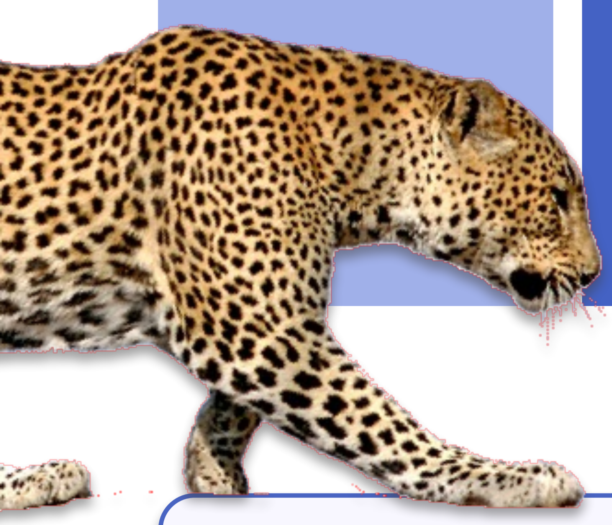
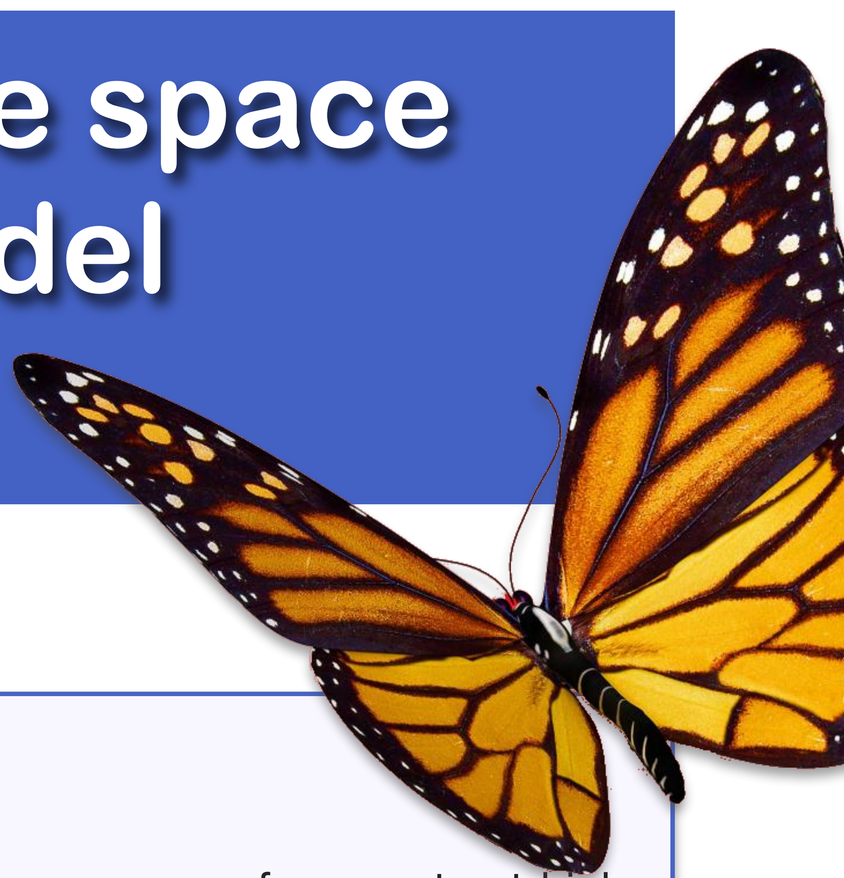


The butterfly diagram: from a phase space portrait to a predator-prey model



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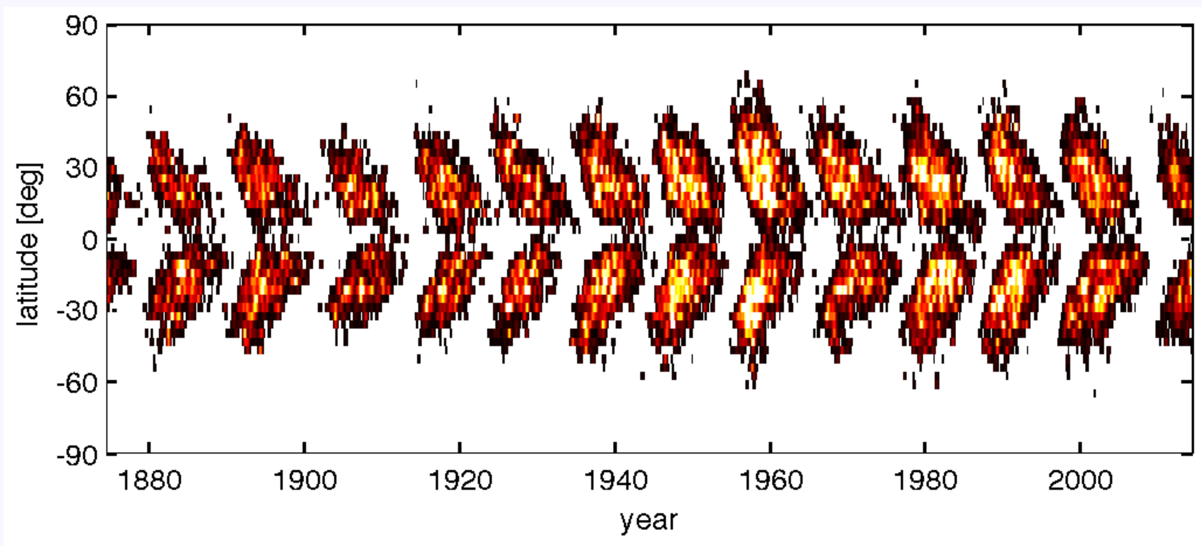
1. The butterfly diagram: still a lot to discover

The latitudinal distribution of the area of sunspots (aka, the *butterfly* diagram) is much more informative than the sunspot number as it reflects the way sunspots migrate under the effect of the solar dynamo.



Cumulated sunspot area vs time and latitude. Colour reflects total area (linear scale)

There have been **many attempts to reduce this spatio-temporal diagram** to sets of simpler proxies of the solar dynamo: projection of spherical coordinates, principal component analysis, etc [Gokhale, Knaack, Hathaway, Mininni, Consolini, …] but their physical interpretation is often debatable.

***Our objective:* use blind source separation to reduce the butterfly diagram to proxies that have a more immediate physical meaning.**

2. Data reduction

Hypothesis: decompose the sunspot areas into separable « modes »

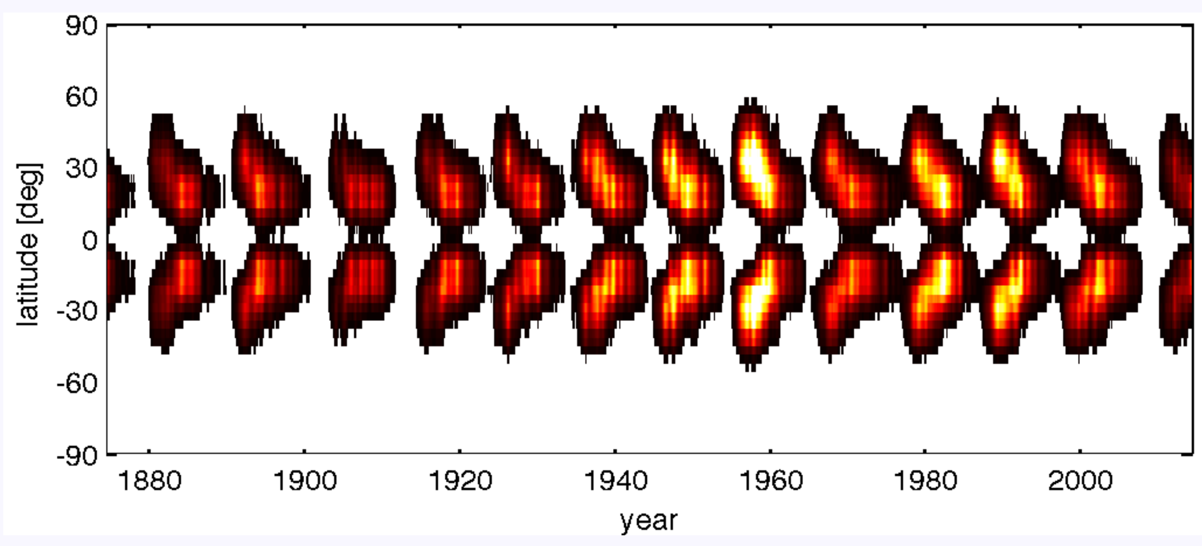
$$A(t,\theta) = \sum_k S_k(\theta) \cdot M_k(t) + \epsilon(t,\theta)$$

sunspot area — **latitudinal profile, or « source »** — **temporal profile, or « mixing coefficient »** — **residual error**

Principal component analysis tells us that all the salient features of the butterfly diagram can be captured by just 2 to 3 modes. But principal components can be negative = no physical meaning.

We apply instead a Bayesian Positive Source Separation [Moussaoui et al., 2002] technique, and constrain the modes with:
- temporal profiles $M_k(t)$ must be independent
- temporal profiles $M_k(t)$ and sources $S_k(\theta)$ must be ≥ 0

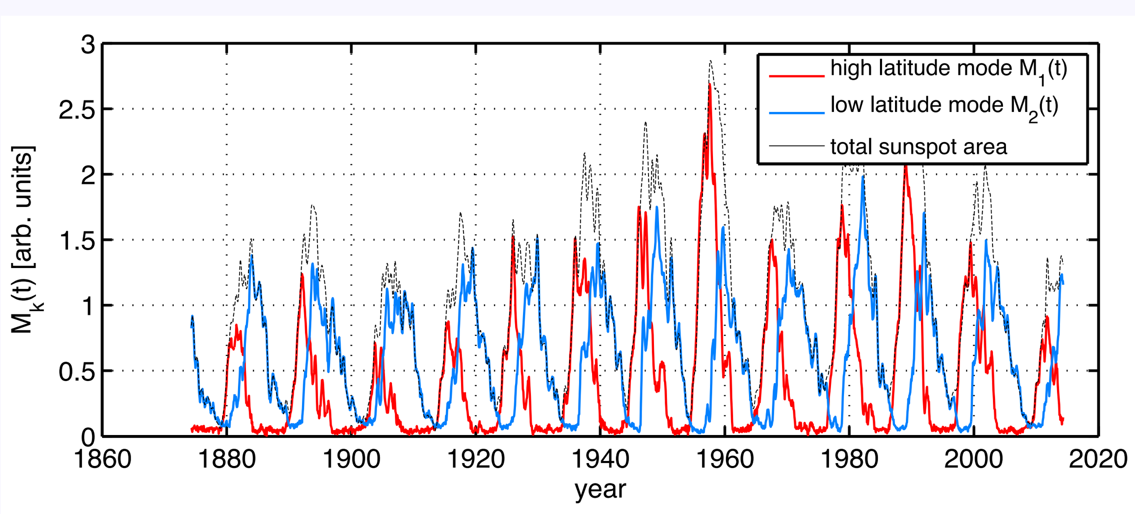
2 sources only suffice to capture all the coherent features of the butterfly diagram. Additional sources merely describe small-scale fluctuations.



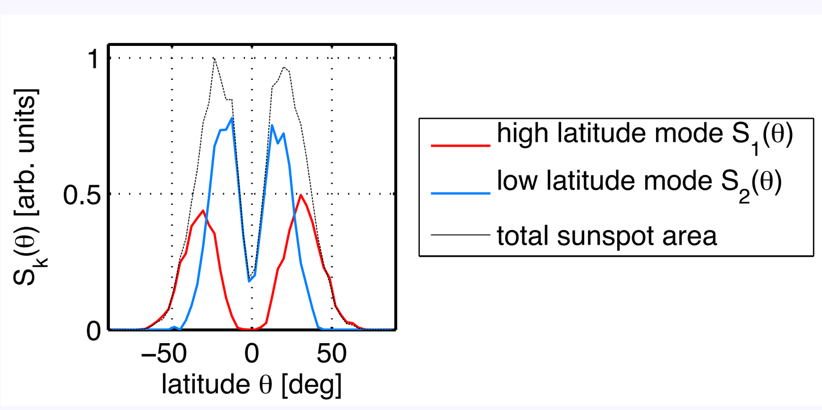
Butterfly diagram reconstructed from two positive sources only.

The key properties of the solar cycle (migration speed, amplitude, …) are now captured by our two temporal profiles $M_1(t)$ and $M_2(t)$.

3. What the modes look like



The two temporal profiles M_1 and M_2.



The corresponding sources, or latitudinal profiles S_1 and S_2.

4. Interpretation

The **high latitude mode $M_1(t)$** describes the emergence of sunspots at high latitudes and is representative of the conversion from poloidal to toroidal flux. This mode directly feeds M_2 = it is the « *prey* »

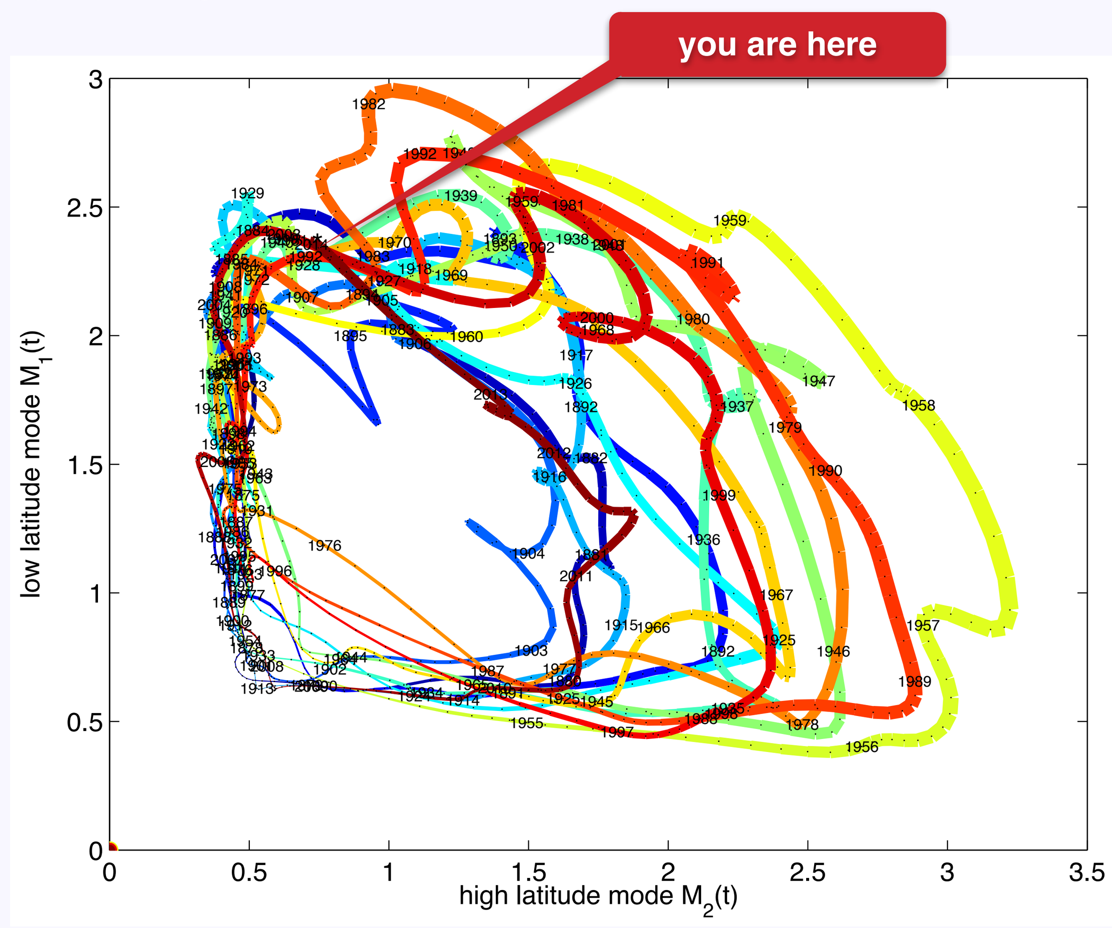
The **low latitude mode $M_2(t)$** describes the disappearance of sunspots at low latitudes and is representative of the conversion from toroidal to poloidal flux. This mode is the « *predator* » of M_1, and subsequently feeds the next cycle.

This representation (re)opens several perspectives

- Describe the asymmetry between both hemispheres and relate this to their synchronisation.
- Asses the evidence for deterministic vs stochastic behavior
- Understand how the characteristics of each cycle (drift speed toward equator, amplitude, duration, etc.) are related.
- Occurrence of Gnevyshev gap at transition from one mode to the other. etc

5. Phase space representation

By plotting M_2 vs M_1 we obtain a concise phase space representation.



Phase space plot. Colour reflects time and line width the total sunspot area. The data have been smoothed over 4 months to ease visualisation.

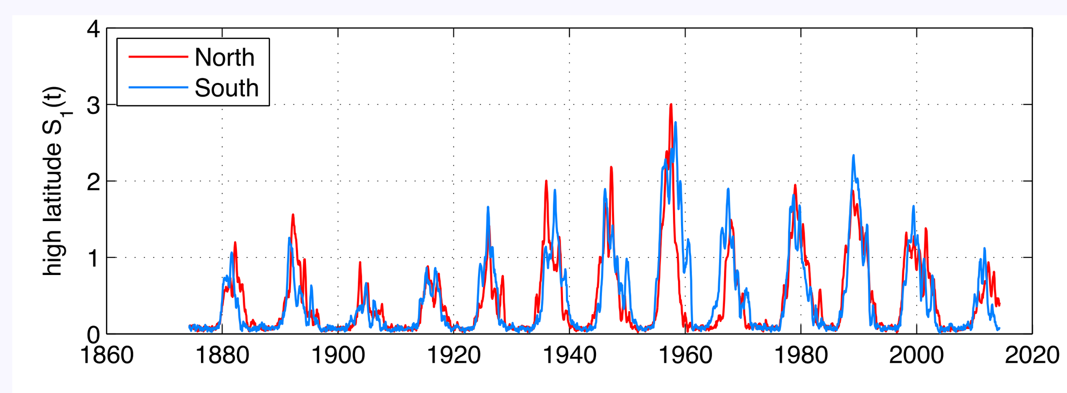
We take beforehand the square root of the sunspot area to stabilise its variance (Anscombe transform)

Interpretation

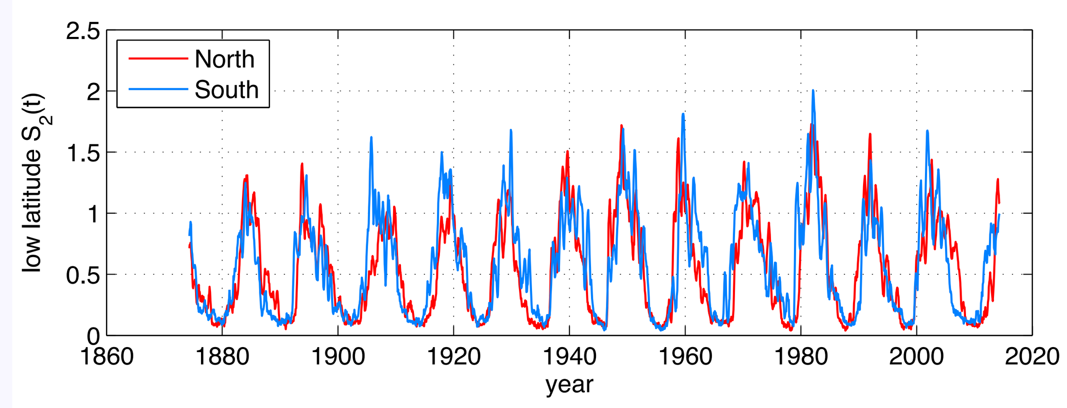
- these orbits are reminiscent of the Lotka-Volterra predator-prey model, which thus gives us a simple analogy of the butterfly diagram.
- two solar cycles are similar **only** of their orbits overlap: we find that the last cycle (nr 23) is analogous to the one that peaked in 1883, and not to the one of 1914, as often suggested.
- this plot gives deep insight in how the transition during sunspot minimum affects the subsequent cycle. More on this soon!

6. Hemispheric asymmetries ?

By estimating the modes separately from both hemispheres, we get a detailed picture of how these asymmetries actually are.



High latitude mode M_1 estimated separately for both hemispheres



Low latitude mode M_2 estimated separately for both hemispheres

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