Turbulence, nonlinear dynamics, and sources of intermittency and variability in the solar wind

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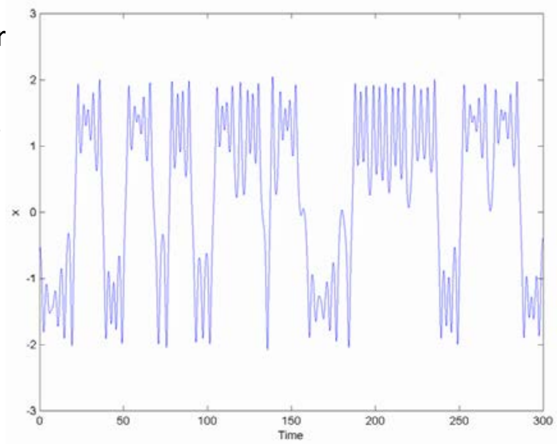
Intermittency & turbulence

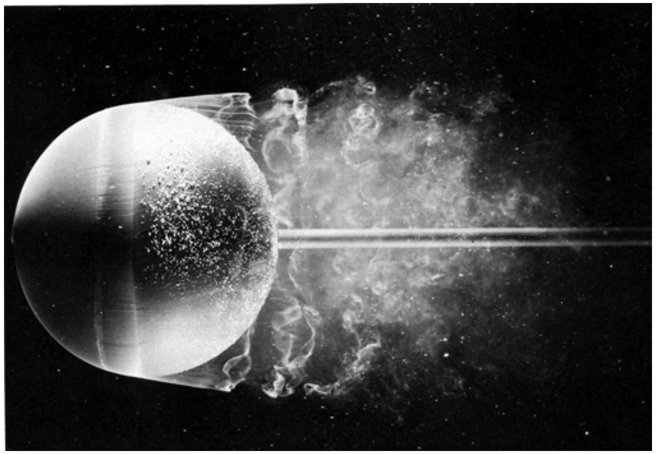
"Intermittency is the nonuniform distribution of eddy formations in a stream. The modulus or the square of the vortex field, the energy dissipation velocity or related quantities quadratic in the gradients of Velocity and Temperature (of the concentration of passive admixture) may serve as indicators. " (E A Novikov, J Appl Math & Nech, 35, 266 (1971)

Intermittency in simple form

- Duffing oscillator
- Lorenz attractor
- Rikitake dynamo
- many others



Spatial fluctuations of dissipation are very large – gradients Are not uniformly distributed; the cascade produces ***intermittency***



Some types of intermittency and potential effects on solar prediction

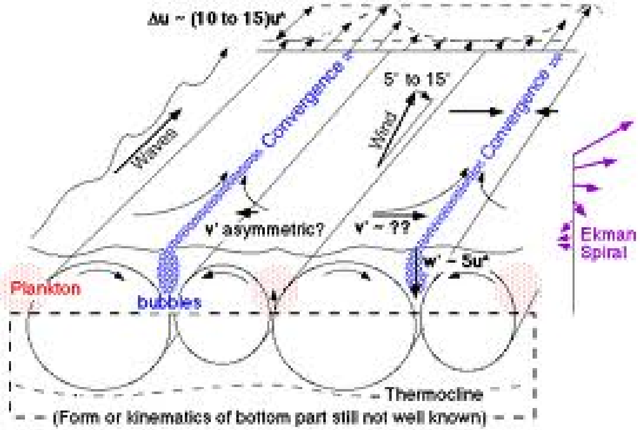
(1) Large scale/low frequency intermittency
  - variability of sources
  - Inverse cascade (space) ←→ 1/f noise (time)
  - Effects of dynamics on the "slow manifold"
  - → Dynamo reversals, rare events (big flares?)

(2) Inertial range intermittency
  - "scaling" range
  - reflects loss of self similarity at smaller scales
  - KRSH
  - → This is a lot of what you see and measure

(1) Dissipation rage intermittency
  - vortex or current sheets or other dissipation structures
  - usually breaks self similarity because there are characteristic physical scales
  - → Controls local reconnection rates and local dissipation/heating;
    small scale "events"

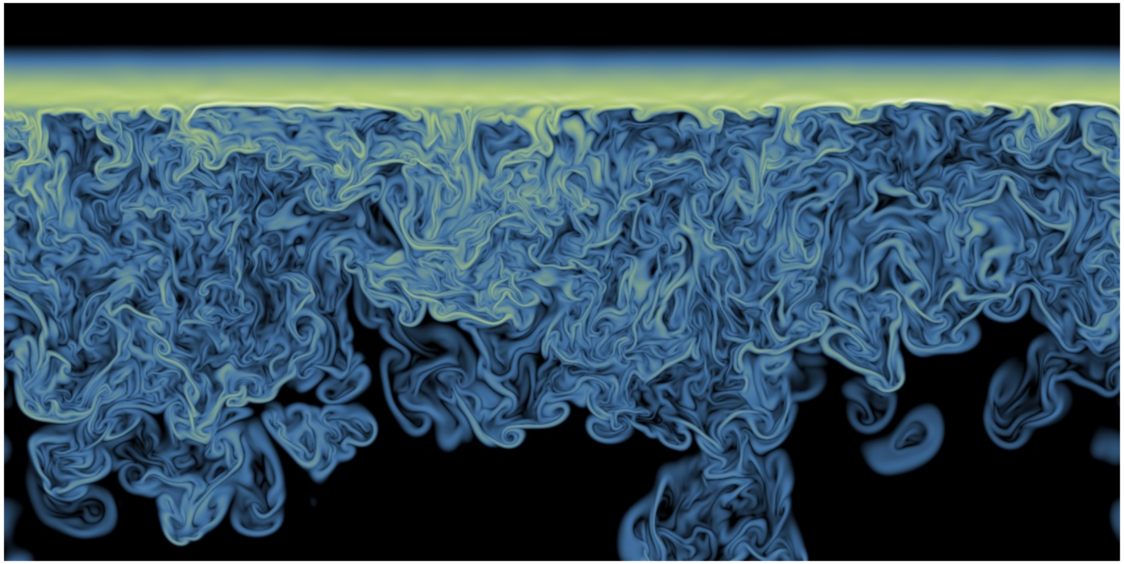
Langmuir cells

- Turbulence
- Waves
- Structure
- Gradients
- Mode coupling

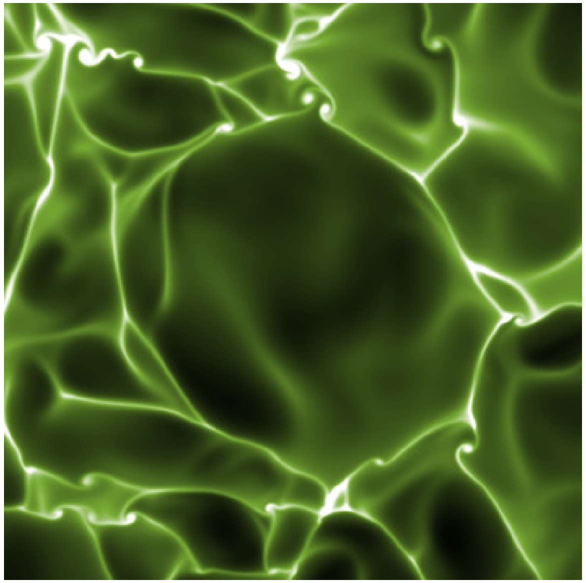


Intermittent turbulence in hydro

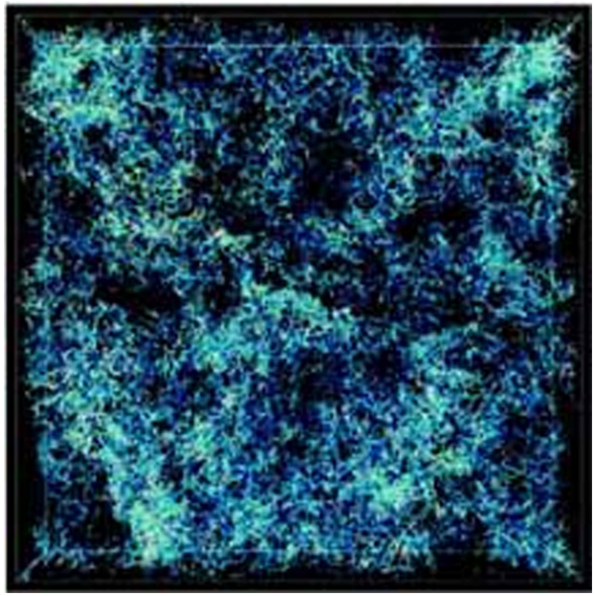
- Dynamics at cloud tops:
temperature gradients,
driven by droplets
(J. P. Mellado, Max Plank Meteor.



- ocean surface-air
Interface (J P Mellado)



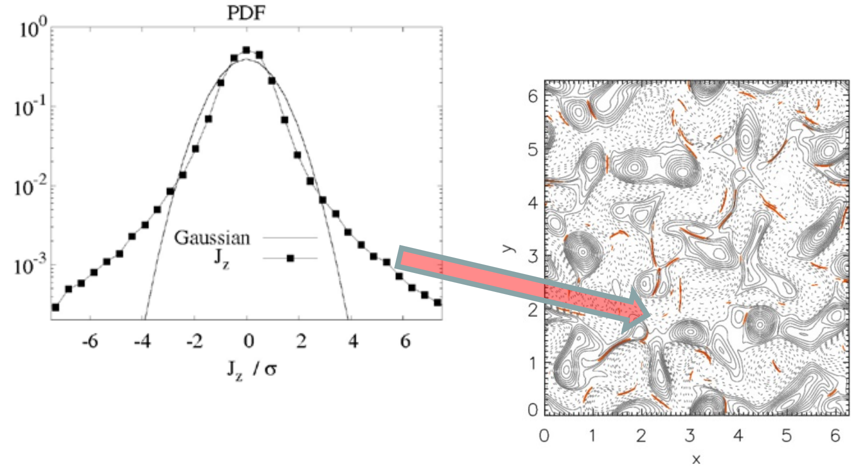
- Vorticity
In interstellar
Turbulence
(Porter,
Woodward,
Pouquet)



- PDFs

intermittency corresponds to "extreme events," especially at small scales

→ fat tails



- Higher order moments and nonGaussianity (esp. increments or gradients)

For Gaussian, odd moments zero,

Even moments $< x^{2n} >$ determined by $<x^2>$;

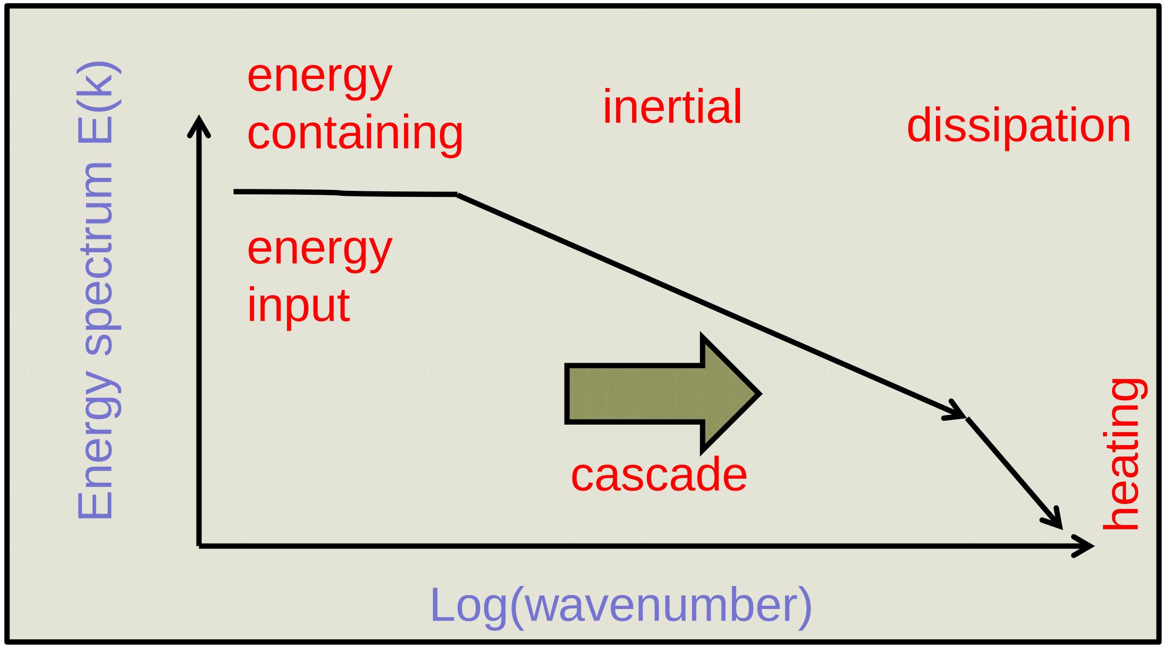
For intermittency, $<x^{2n}>$ > Gaussian value

- Kurtosis and filling fraction F

$\kappa = <x^4>/ <x^2>^2$

HEURISTIC: $\kappa \sim 1/F$

“standard” turbulence spectrum



- **Dissipation:** conversion of (collective) fluid degrees of freedom into motions into kinetic degrees of freedom
- **Heating:** increase in random kinetic energy
- **Entrop**y increase: irreversible heating

How nonlinearity and cascade produces intermittency

Concentration of gradients

- Amplification of higher order moments
  - Suppose that q & w are Gaussian, and $\frac{dq}{dt} \sim qw$ then pdf($\frac{dq}{dt}$) is exponential-like with κ ▯ 6-9.
- Amplification greater at smaller scale ($\mathrm{e.g.}$, $\frac{dq}{dt} \sim kqw$,wavenumber k)
- Role of stagnation points (coherency!)
  - No flow or propagation to randomize the concentrations
- Formation is *IDEAL* *(e.g., Frisch et al. 1983; Wan et al, PoP 2009)*
- Dissipation is more intense in presence of gradients → relation between intermittency and dissipation

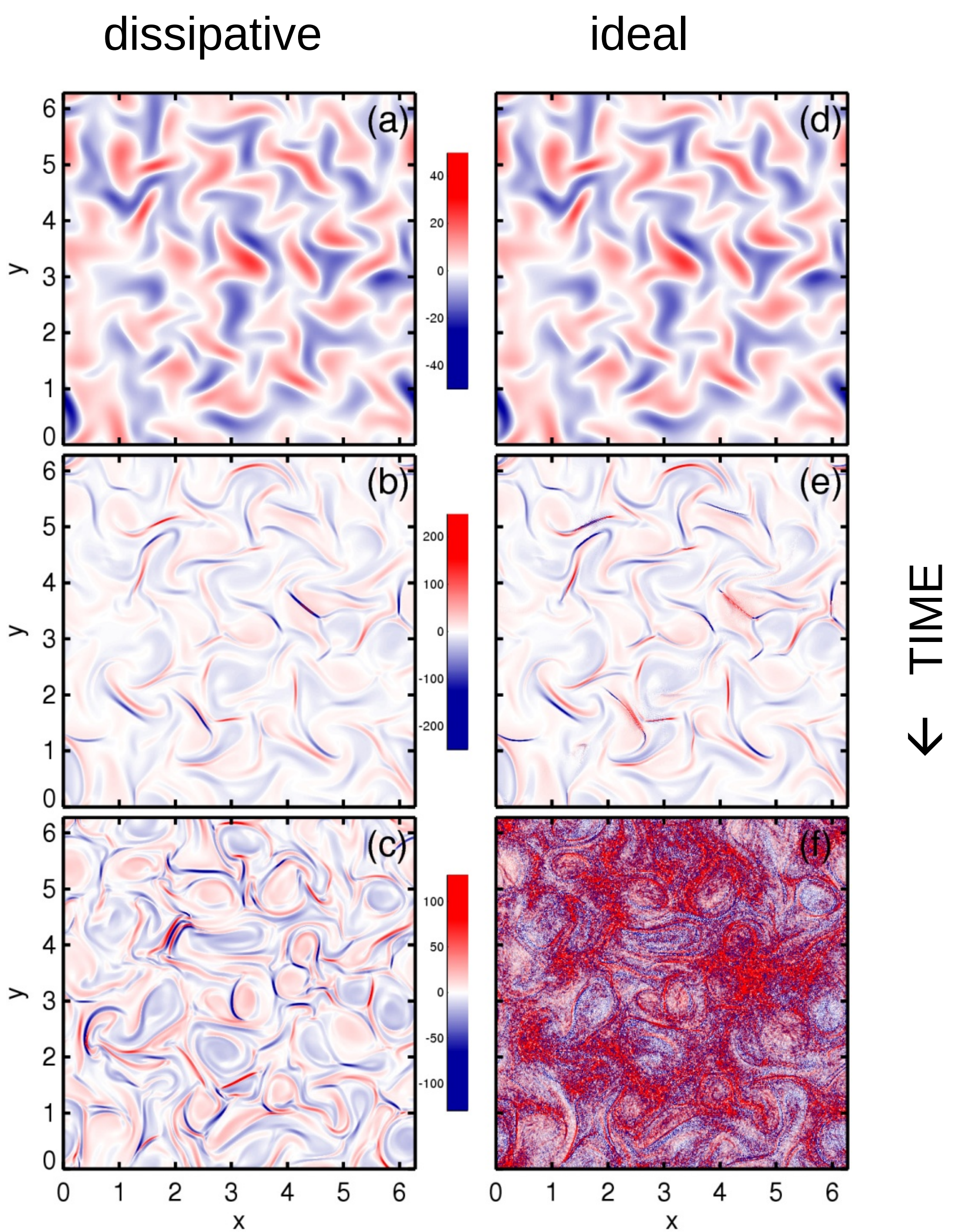
Coherent structures are generated by ideal effects!

Contours of current density:

Same initial condition →

Already have non-Gaussian coherent structures →

…before finite resolution errors set in →



Turbulent fluctuations have structure and dissipation is not uniform

Kolmogorov '41 ε *: dissipation rate;* Δv_r*: velocity increment*

$$\Delta v_r \sim (\varepsilon r)^{1/3}$$

$$\rightarrow \langle \Delta v_r^{\,p} \rangle = \text{const.}\ \varepsilon^{p/3}\, r^{p/3}$$

But this is NOT observed!

Kolmogorov '62 $\varepsilon_r = r^{-3} \int_r d^3x'\, \varepsilon(x')$

$$\Delta v_r \sim (\varepsilon_r\, r)^{1/3}$$

Kolmogorov refined similarity hypothesis

$$\rightarrow \langle \Delta v_r^{\,p} \rangle = \text{const.}\ \langle \varepsilon_r^{\,p/3} \rangle\, r^{p/3}$$

$$= \text{const.}\ \varepsilon^{p/3}\, r^{p/3 + \xi(p)}$$

(Oubukhov '62)
→multifractal theory comes from this!

Intermittency in hydrodynamics • Anselmet et al, JFM 1984

Velocity structure functions in turbulent shear flows 69

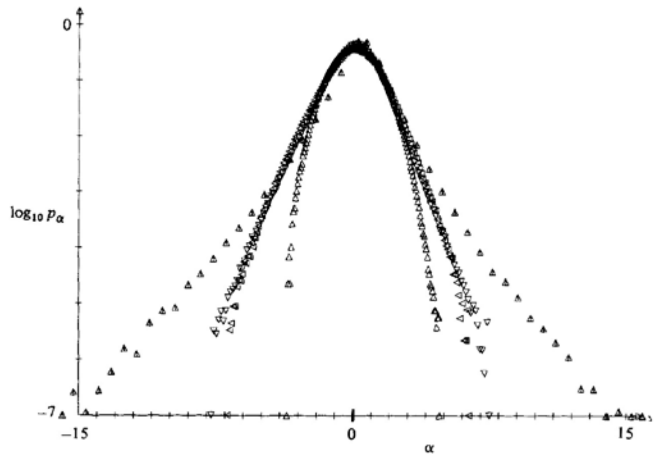


FIGURE 1. Probability density functions in the axisymmetric jet at $R_\lambda = 536$ of u and Δu normalized by their respective standard deviations, $\alpha = \Delta u / \langle (\Delta u)^2 \rangle^{\frac{1}{2}}$: △, $r = 0.6$ mm $= 3.5\eta$; ▽, 7.7 mm; ◁, 17.2 mm. △, $\alpha = u / \langle u^2 \rangle^{\frac{1}{2}}$.

Need to be sure
Pdf is resolved well
enough to compute
higher order moments!

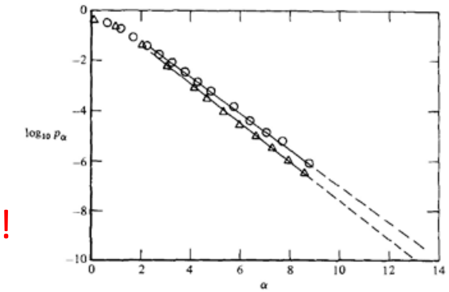


FIGURE 3. Probability density function of $\Delta u / \langle (\Delta u)^2 \rangle^{\frac{1}{2}}$ for $r \approx \lambda$ in axisymmetric jet at $R_\lambda = 536$: ○, $\alpha = \Delta u / \langle (\Delta u)^2 \rangle^{\frac{1}{2}} < 0$; △, $\alpha > 0$. Broken lines are extrapolations of solid lines beyond the experimental range of α.

Pdfs of longitudinal velocity increments have fat tails; fatter for smaller scales

$$\langle \Delta u_r^n \rangle \sim r^{\zeta(n)}$$

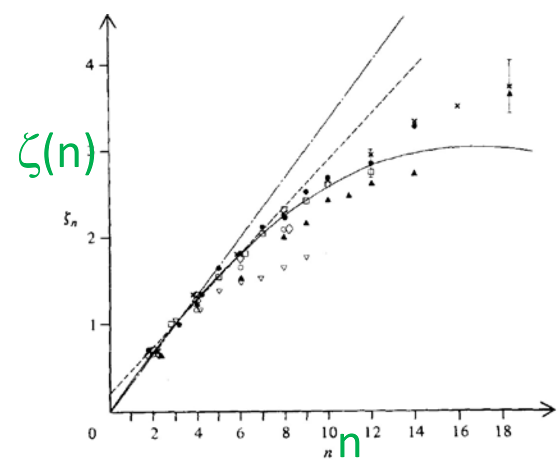


FIGURE 14. Variation of exponent ζ_n as a function of the order n. ●, $R_\lambda = 515$ (duct); □, 536; ×, 852. Symbols ○, ▲, ▽, ◇ are respectively the exponents given by Mestayer (1980); Vasilenko *et al.* (1975); Van Atta & Park (1972); and Antonia *et al.* (1982*a*). The solid curve is LN with $\mu = 0.2$, the dotted curve the β-model and the chain-dotted line Kolmogorov's (1941) model.

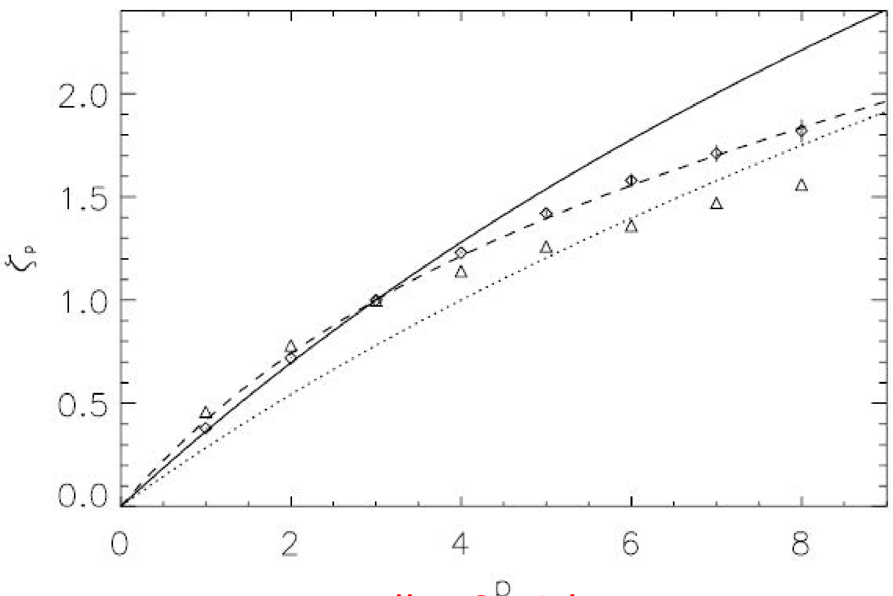
Scaling of exponents at increasing order: reveals departures from self similarity and multifractal scalings (beta, log-normal, She-Levesque, etc

SW/MHD intermittency

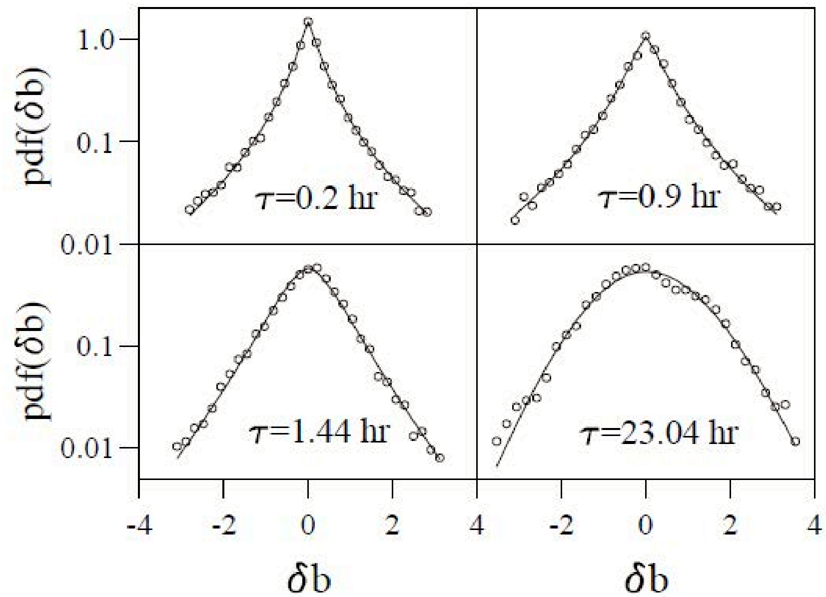
- More dynamical variables
- Analogous effects

Intermittency in MHD & Solar wind

- Multifractal scalings (Politano et al, 1998; Muller and Biskamp 2000)
- PDFs of increments (Burlaga, 1991; Tu & Masrch 1994, Horbury et al 1997)



Muller & Biskamp, 2000



Sorriso et al, 1999

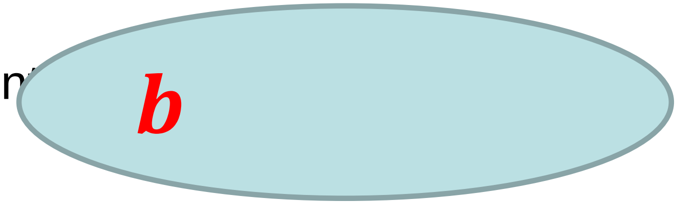
Cellularization, turbulent relaxation and structure in plasma/MHD:

large scale evolution produces local relaxation → suppression of nonlinearity → nonGaussian statistics → boundaries of relaxed regions correspond to small scale intermittent structures

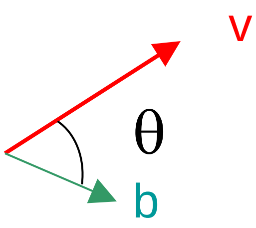
- *Local* relaxation can give rise to
    - Force free states
    - Alfvenic states
    - Beltrami states

AND

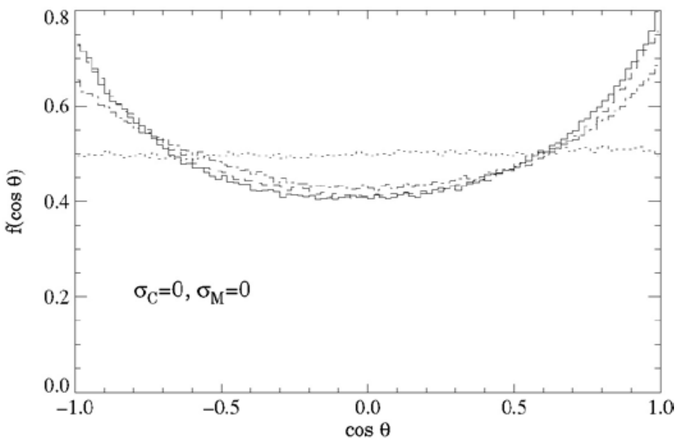
- characteristic small scale intermitten
structures , e.g. current sheets



- Simulations show RAPID relaxation & production of local correlations.
- Spatial "patches" of correlations bounded by discontinuities.



Characteristic distributions appear in *less than one nonlinear time!*



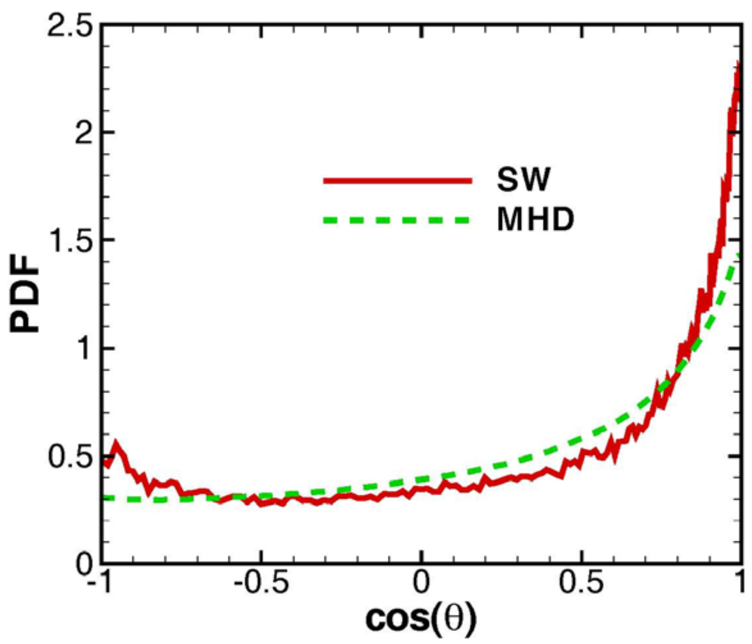
Directional alignment: pdf f(cos(vb))

Run with Hc 0

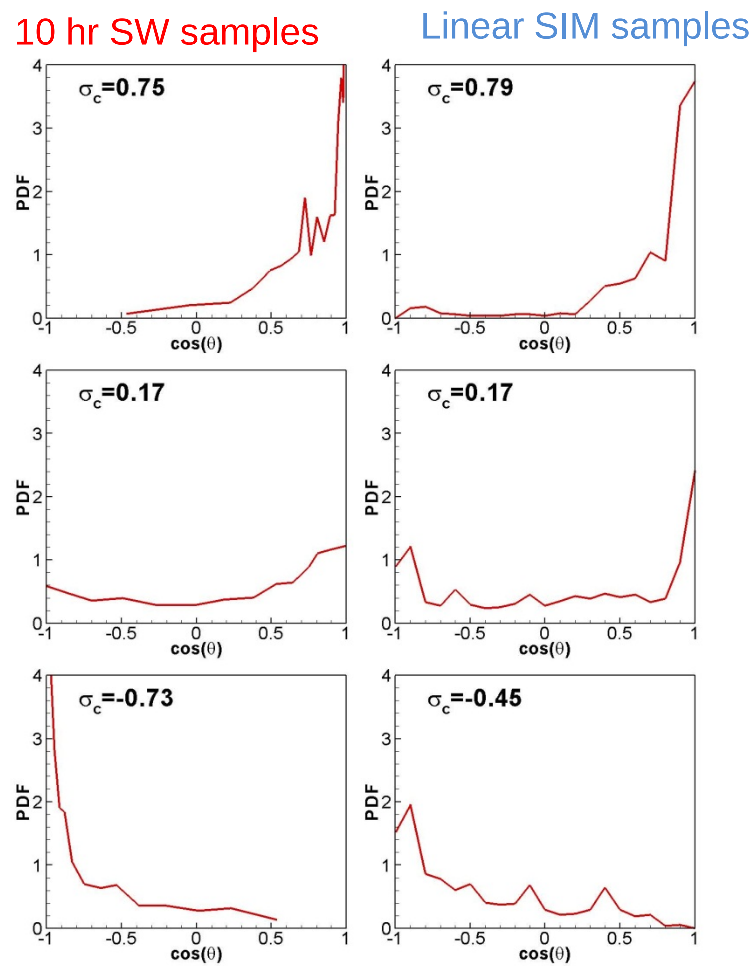
v-b correlations: large (black >0; white < 0)
(here, 2D MHD)

• Analysis of patches of Alfvenic correlations

- Distributions of $\cos(\theta)$ [angle between velocity and magnetic field]
- Global statistics & statistics of linear subsamples (~1-2 correlation scales)
- SW and 3D MHD SIM (512^3)
- Global Alfvenicity $\sigma_c \approx 0.3$



- For a specified sample size, can get highly variable Alfvenicity (see Roberts et al. 1987a,b)
- Same effect in SW and in SIMs!



PVI Coherent Structure Detection: designed to work the same way in analysis of solar wind and simulation data

-

$$PVI = \frac{|\Delta \boldsymbol{B}(\boldsymbol{x,s})|}{<|\Delta \boldsymbol{B}(\boldsymbol{x,s})|^2>^{1/2}}$$

Δ (x;s) = (x + s) – $\boldsymbol{B}$(x)

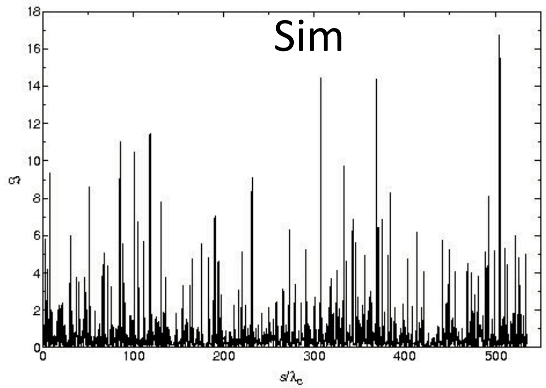
-
-
-

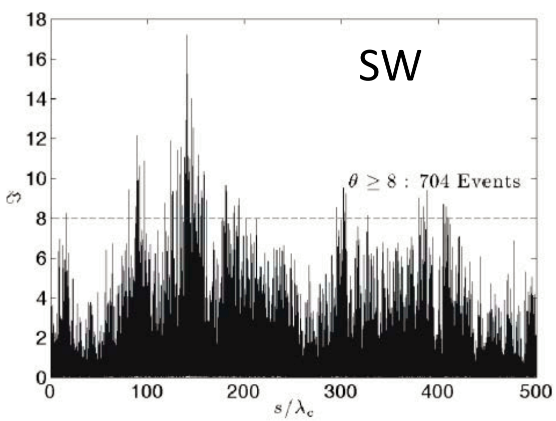
→

Greco et al, GRL 2008; ApJ 2009

PVI links classical discontinuities and intermittency & compares well between SW and simulations

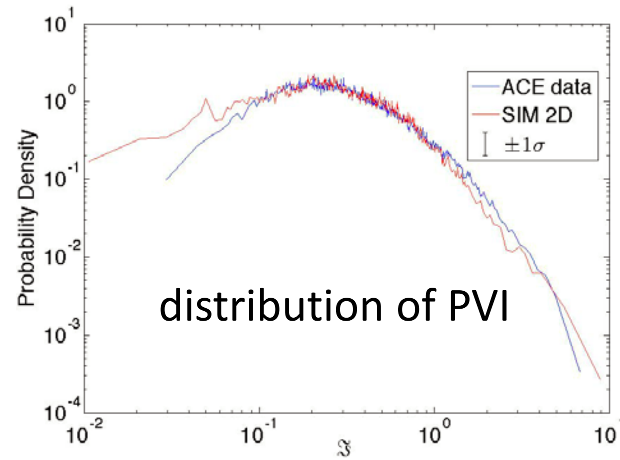
Greco et al, ApJ 2009; Servidio et al JGR, 2011



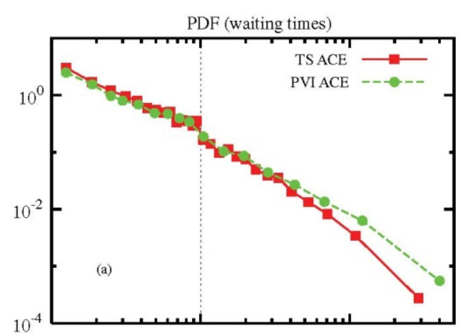


500 correlation scale PVI time series

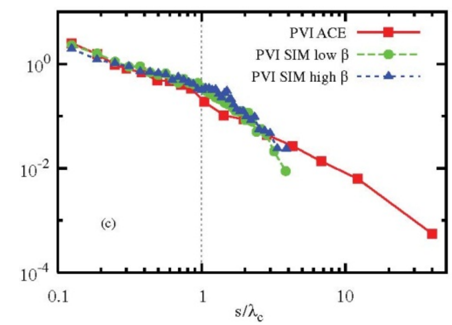
PVI > 3 events are statistically inconsistent with Gaussian statistics at the 90+ % level



Waiting time distribution between "events"



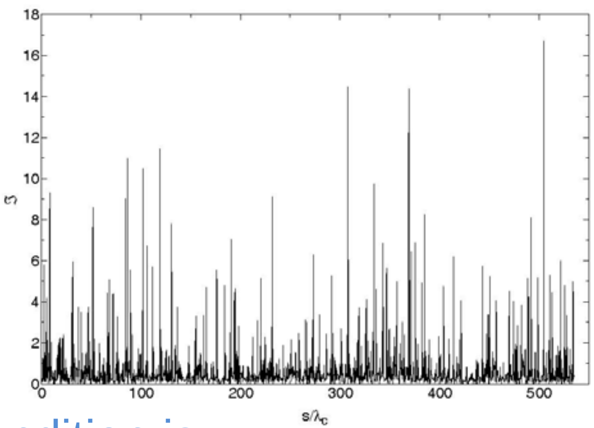
PVI vs. classical discontinuity methods



PVI events in SW And in MHD turbulence simulations

- Use PVI to find reconnection sites
- In SIMs & in SW (caveats)

From Servidio et al, JGR, 116, A09102 (2011)

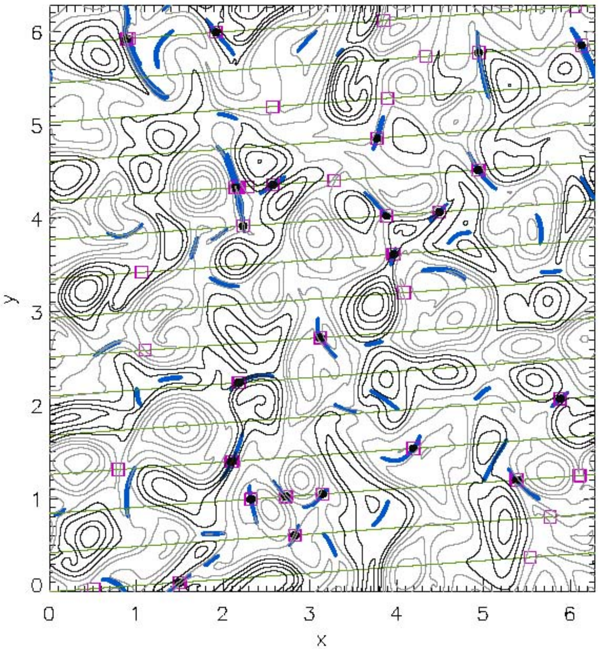


Condition is
PVI > threshold (1,2,3…

Trajectory thru SIM →

←"time series" of PVI

↓ Get a
Table of efficiencies



At PVI>7
- only ID ~40% of reconnection sites
- But >95% of events are reconnection sites

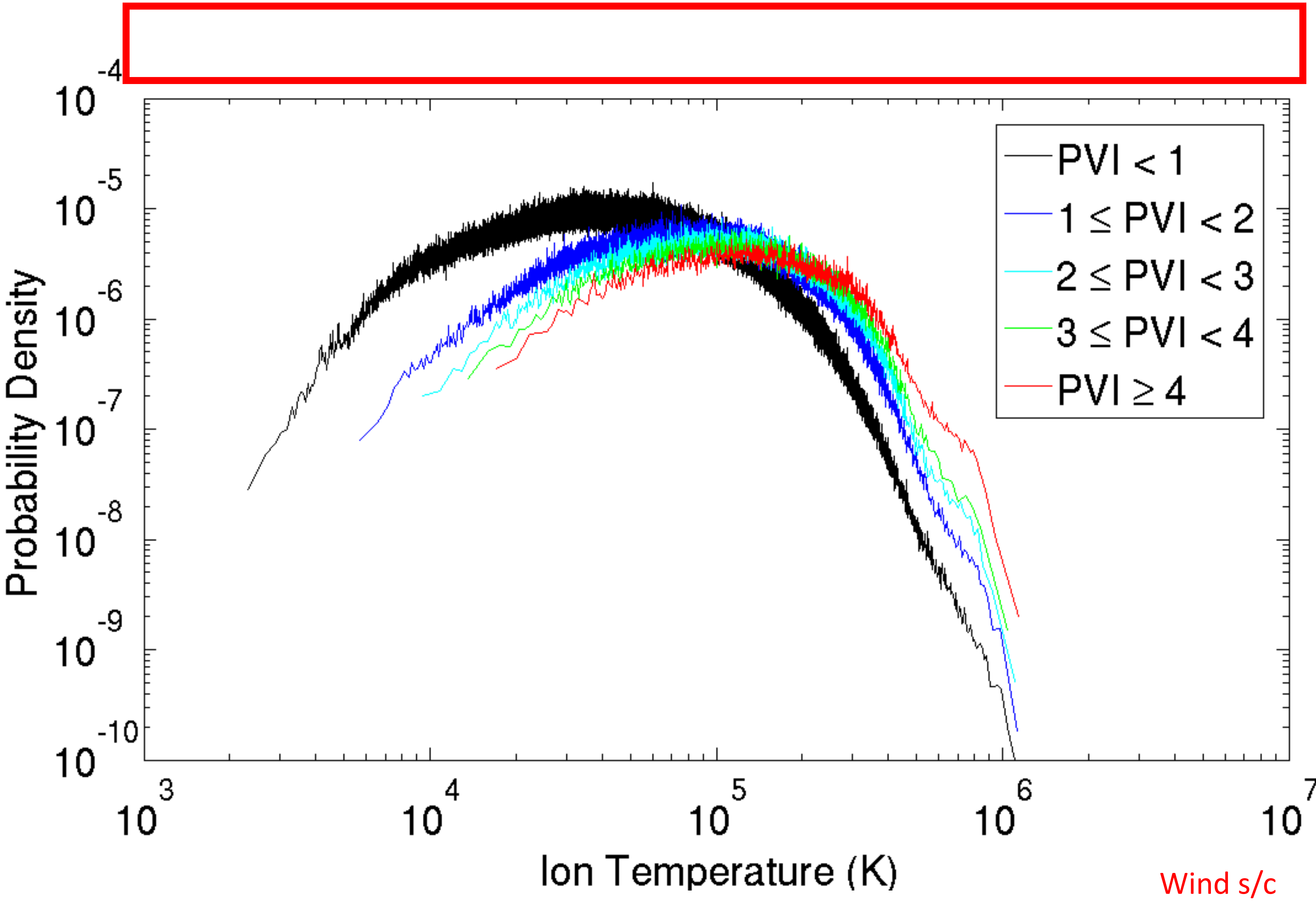
Same approach in SW, but compare t Gosling/Phan identified exhaust events:

PVI>7 event in SW very likely to be at/near a reconnection event!

Osman et al, PRL 112, 215002 (2014)

Evidence that coherent structure are sites of enhanced heating:
Solar wind proton temperature distribution conditioned on

$$PVI = \frac{|\Delta \mathbf{B}(x,s)|}{<|\Delta \mathbf{B}(x,s)|^2>^{1/2}}$$

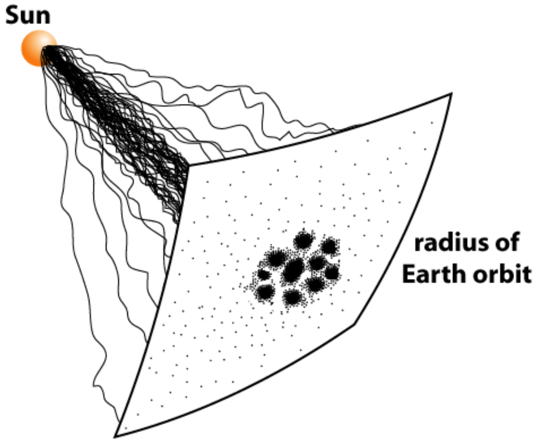


Similar (weaker) effects in: electron temp & electron heat flux

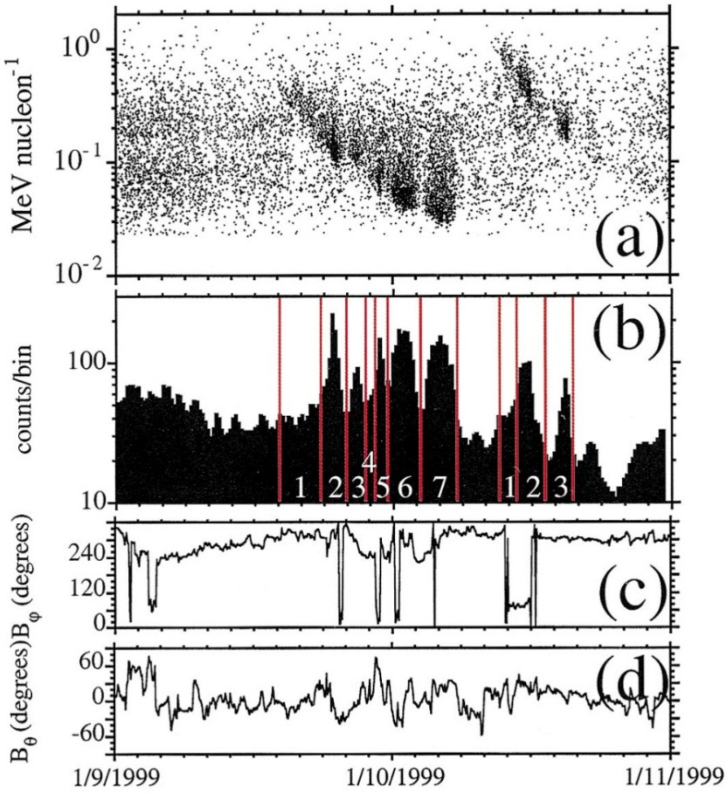
ALSO: neighborhoods of larger PVI events are hotter

Wind s/c
Osman et al, 2011
Osman et al, 2012

Implications for energetic particle transport



Transport boundaries are observed: "dropouts" of Solar energetic particles



H-FE ions vs arrival time
For 9 Jan 1999 SEP event
From Mazur et al, ApJ (2000)

plasma intermittency

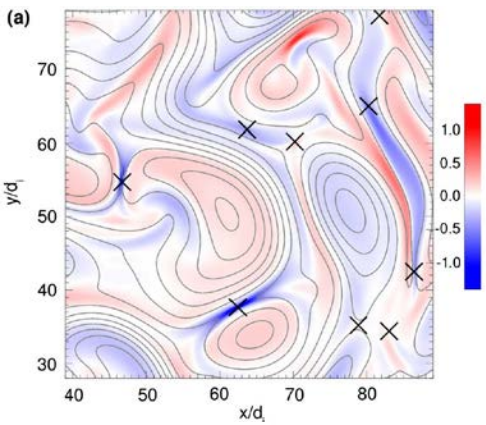
- At kinetic scales
- Still more variables, but analogous effects

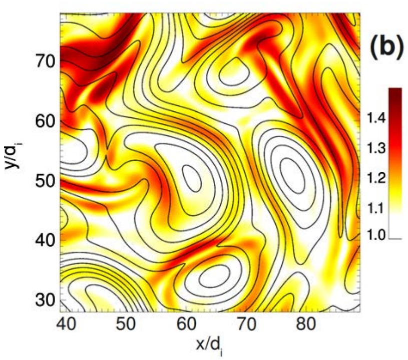
Localized kinetic effects in 2.5D Eulerian Vlasov simulation (undriven initial value problem; strongly turbulent)

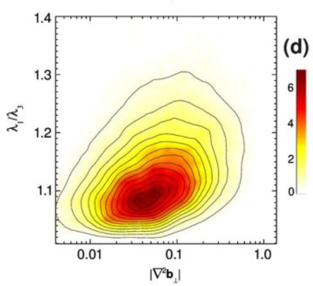
- Magnetic field, current density, X points

Servidio et al, PRL 108, 045001 (2012)

Out-of-plane current



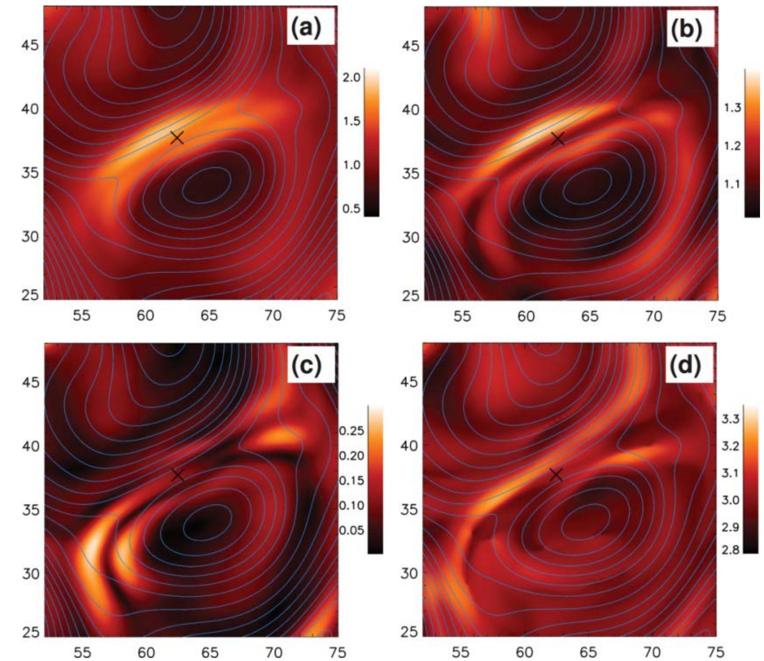




- Anisotropy Tmax/Tmin in small area

- Kinetic effects near a “PVI event”

Greco et al, PRE, 86, 066405 (2012)

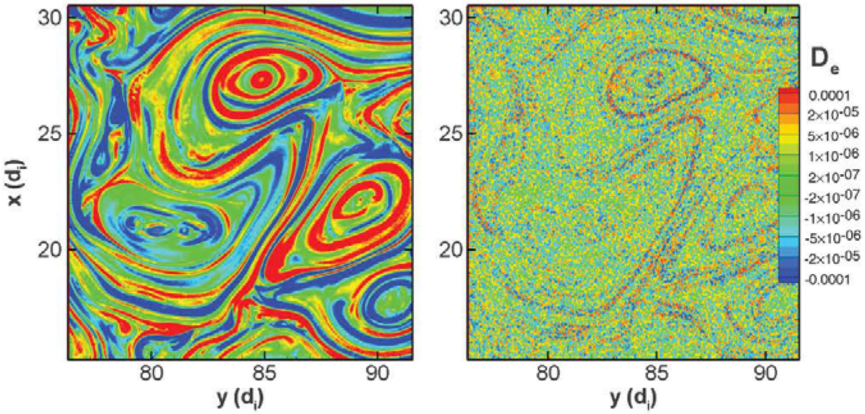


- a) nonMaxwellianity
- b) proton T anisotropy
- c) proton heat flux
- D) kurtosis of f(v)

There is a strong association of kinetic effects with current structures!

Dissipation is concentrated in sheet-like structures in kinetic plasma

PRL **109,** 195001 (2012) PHYSICAL REVIEW LETTERS week ending 9 NOVEMBER 2012



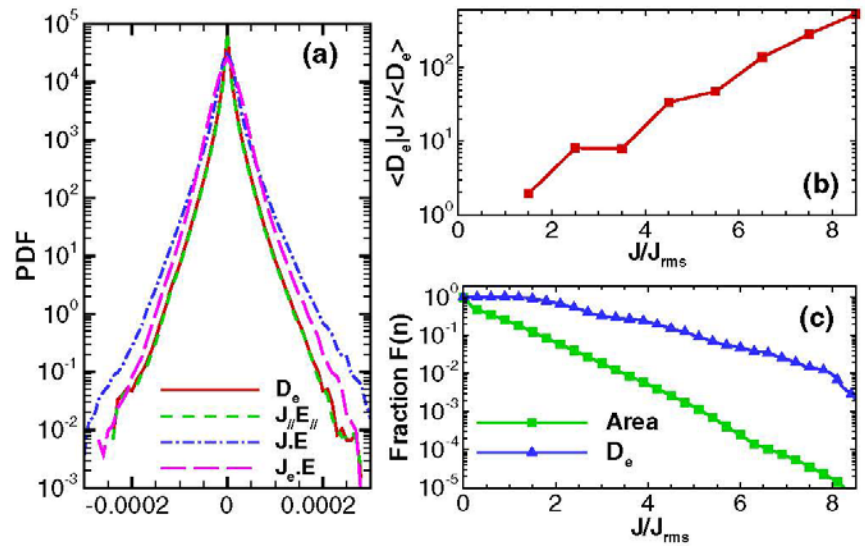


FIG. 2 (color). (Left) J_z in a close-up region of the simulation domain showing hierarchy of coherent structures; (right) Contour of electron-frame dissipation D_e for the region shown

Wan, Matthaeus, Karimabadi, Roytershteyn, Shay, Wu , Daughton, Loring, Chapman, 2012

Strength of electric current density in shear-driven kinetic plasma (PIC) simulation (see Karimabadi et al, PoP 2013)

Thinnest sheets seen are comparable to electron inertial length. Sheets are clustered
At about the ion inertial length → heirarchy of coherent, dissipative structures at kinetic scales

Scale dependent kurtosis:
MHD, kinetic sims, SW comparison

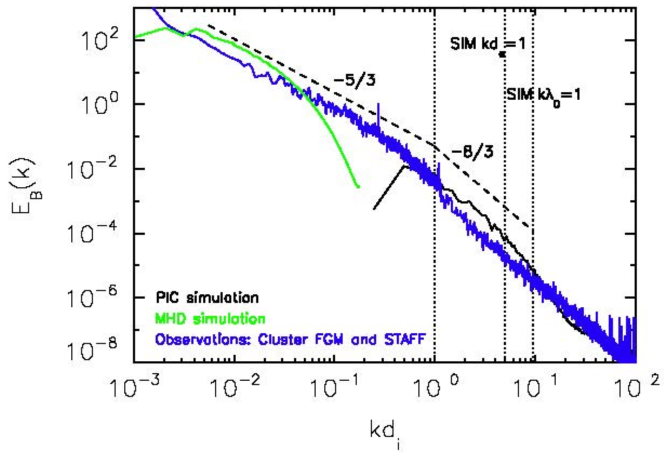


Figure 1. Magnetic energy spectrum from P3D simulation with wavenumber scaled to ion inertial scale kd_i (first vertical dashed line); also shown—for PIC case only—are electron skin depth $kd_e = 1$ and Debye scale $k\lambda_D = 1$. For qualitative comparison, spectra from Cluster FGM and STAFF (only $kd_i = 1$ is relevant), and MHD simulation (d_i associated to 1/10 Kolmogorov dissipation scale) are also shown.

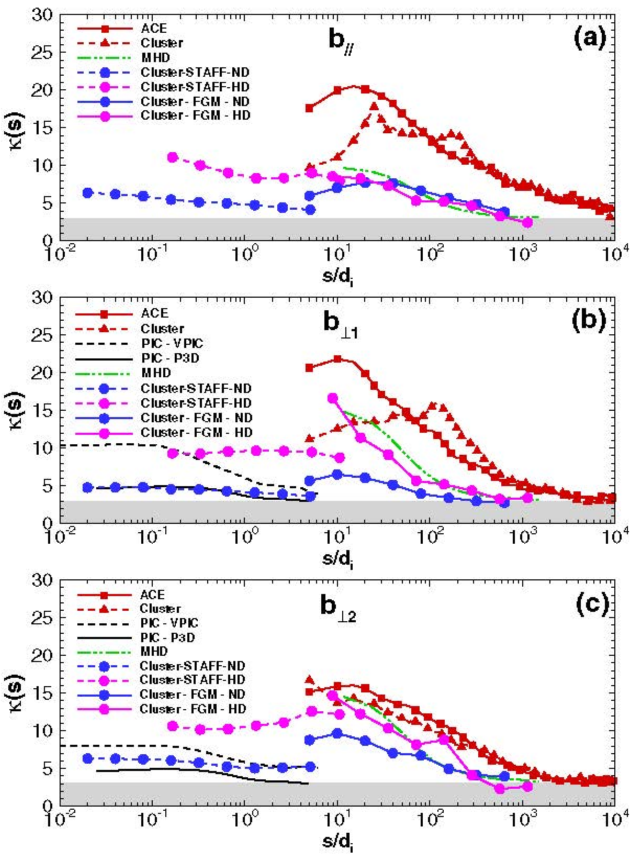


Figure 2. Kurtosis of magnetic field increments $\kappa(s)$ vs. s for three components of magnetic field in mean field coordinates: (a) $b_{\parallel}$, (b) $b_{\perp 1}$, and (c) $b_{\perp 2}$, where $b_{\parallel}$ is the component in the mean magnetic field direction and $b_{\perp 2}$ is perpendicular to the mean magnetic and velocity field. Spatial lag s normalized to d_i is set to one-tenth of the dissipation scale for the MHD case. At smaller scales, $\kappa(s)$ is computed from PIC simulations ("VPIC" and "P3D") and Cluster STAFF normal-density ("STAFF-ND") and Cluster STAFF high-density ("STAFF-HD") intervals. At large scales, $\kappa(s)$ is computed from MHD simulation, ACE data, and long-time (4 hr) Cluster FGM data ("Cluster"). In addition, shorter FGM intervals probe correspondence with STAFF data in similar intervals.

Wu et al, ApJ Letters
763:L302012 (2013)

Very low frequency/very large scale intermittency

- 1/f noise:
  - Gives “unstable” statistics – bursts and level-changes
  - Long time tails on time correlations
  - Generic mechanisms for its production (Montroll & Schlesinger, 1980)
  - Often connected with inverse cascade, quasi-invariants,
  - highly nonlocal interactions (opposite of Kolmogorov’s assumption!)

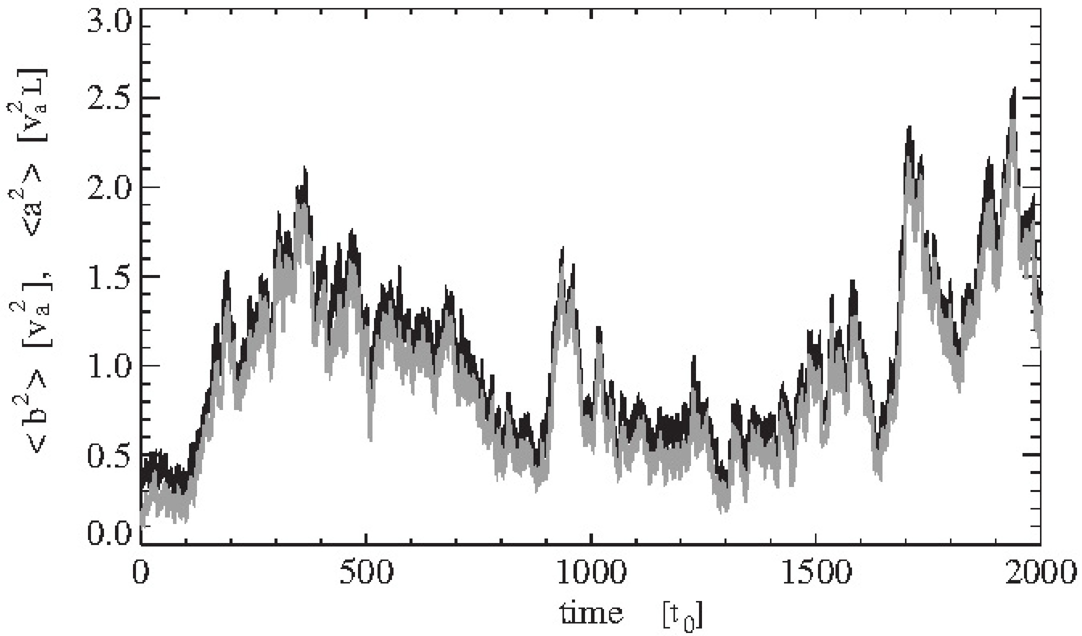
- Dynamo generates 1/f noise (experiments: Ponty et al, 2004
    - connected to statistics of reversals (Dmitruk et al, 2014)
  - 1/k → 1/f inferred from LOS photospheric magnetic field
  - 1/f signature in lower corona
  - 1/f signatures observed in density and magnetic field in solar wind

  at 1 AU (M+G, 1986; Ruzmaiken, 1988; Matthaeus et al, 2007; Bemporad et al, 2008)

An example from 3D MHD with strong mean magnetic field

(Dmitruk & WHM, 2007)

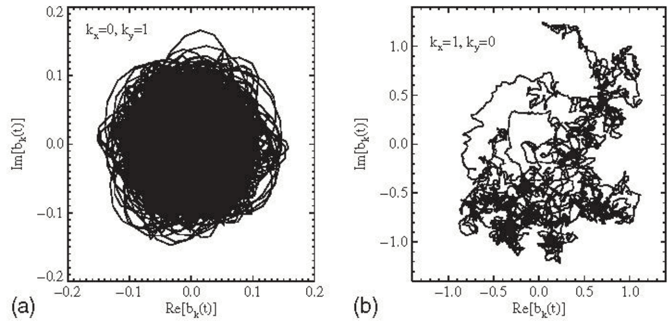
- nearly in condensed state
- energy shifts at times scales of 100s to 1000s Tnl
- characteristic Tnl ~ 1
- Where do these timescales come from ?



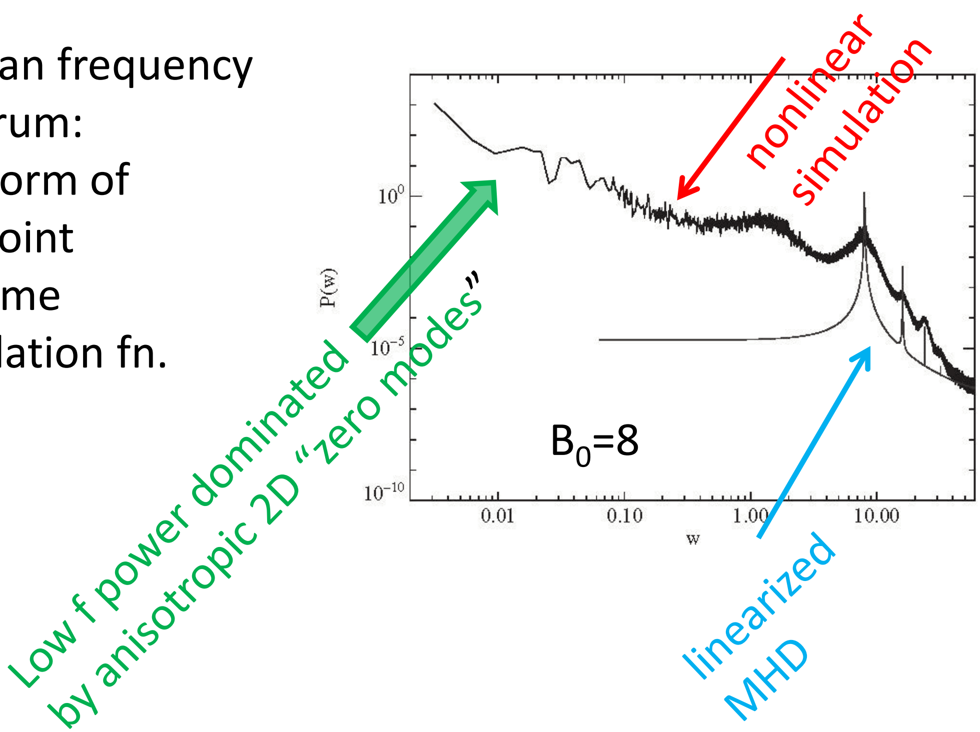
Numerical experiments on MHD Turbulence with mean field: onset of 1/f noise due to “quasi-invariant”

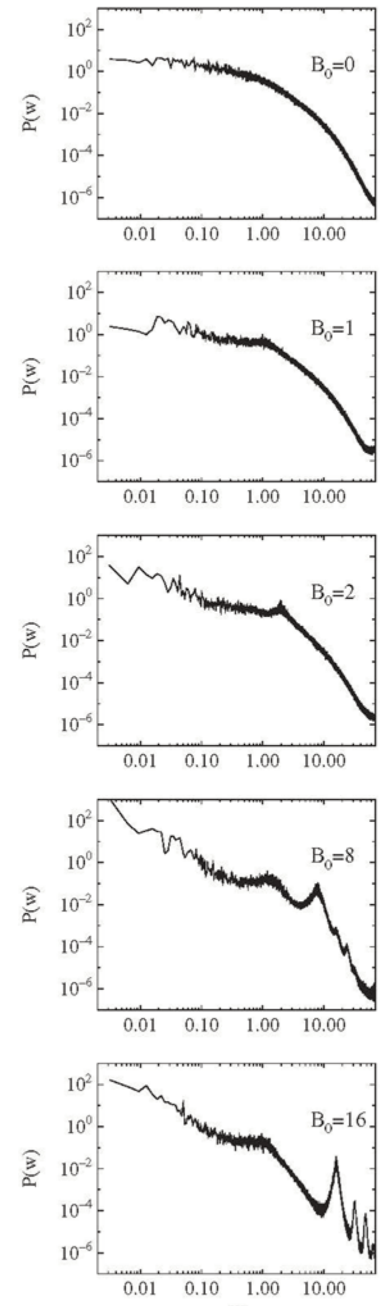
Eulerian frequency spectra

behavior of
a Fourier mode
In time, from
simulation



Eulerian frequency
spectrum:
transform of
one point
two time
Correlation fn.

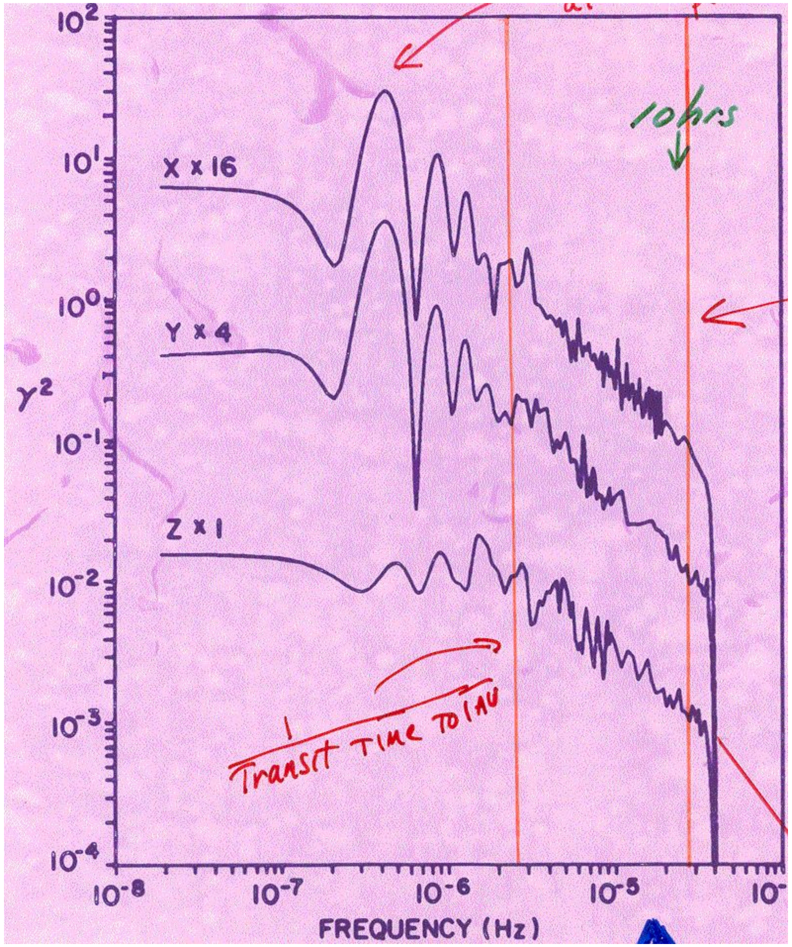


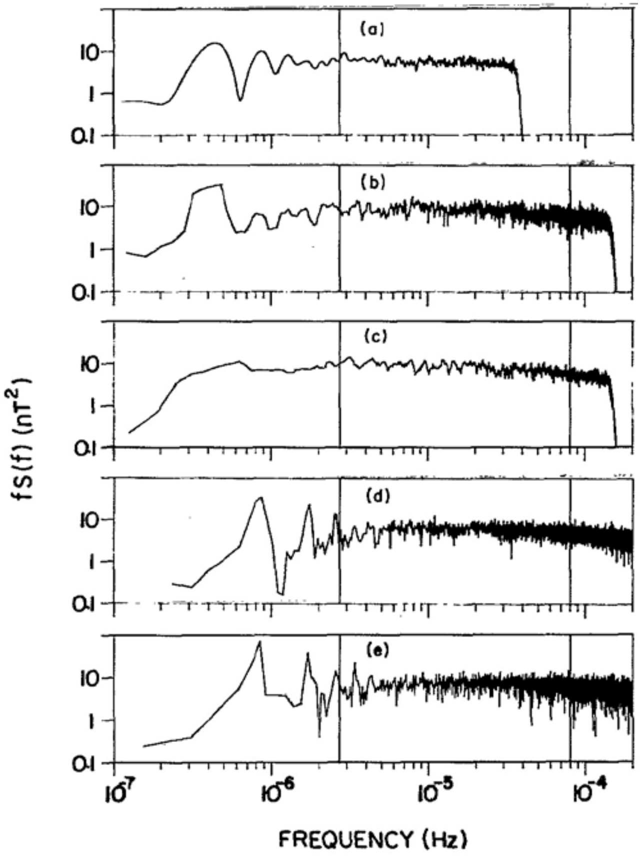


Increasing B_0

Dmitruk & Matthaeus, 2009

1/f noise in SW (1AU ISEE-3, OMNI datasets)





Matthaeus & Goldstein, PRL 1986

1/f: 1AU, MDI and UVCS – high/low latitude comparisons

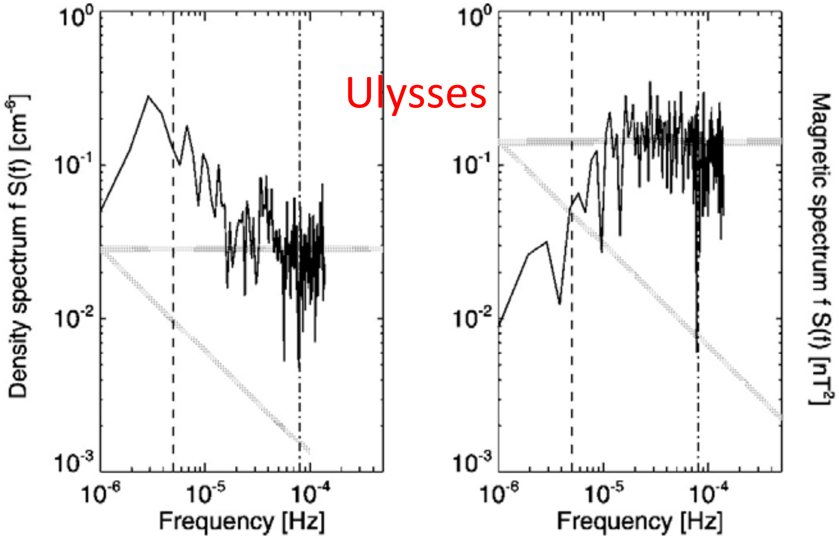
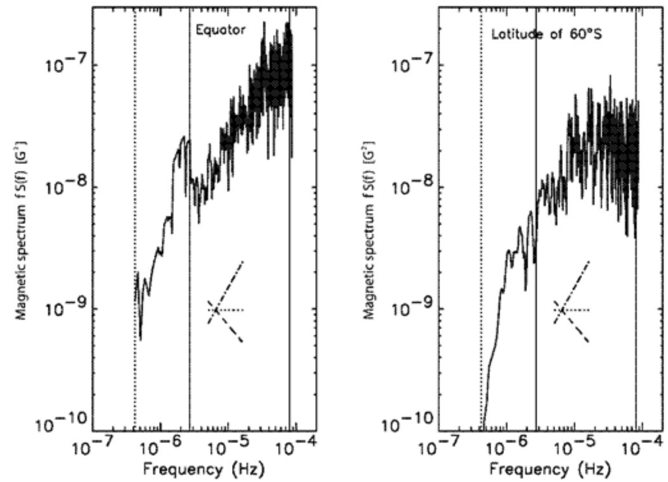
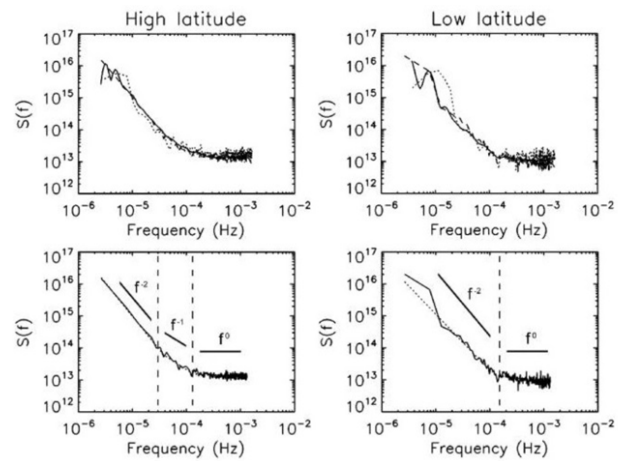


FIG. 1.—Examples of compensated spectra, $fS(f)$, showing intervals of $1/f$ noise, in magnetic field (*right*) and density (*left*) , from *Ulysses* data at low latitude, near 43° latitude, for days 116–176, 1996. Vertical dashed lines indicate the approximate frequency range of $1/f$ noise reported by Matthaeus & Goldstein (1986). Shaded bars suggest $fS(f) \sim f \times 1/f$ variation (flat), and, for reference, $fS(f) \sim f \times 1/f^{5/3}$ "Kolmogoroff" variation.



MDI



UCVS

FIG. 3.—*Top*: Lyα power spectra $S(f)$ (photons^2 cm^{-4} s^{-2} sr^{-2}) from FT (*dotted lines*), WT (*solid lines*), and LS (*dashed lines*) analyses averaged over a 10° latitude interval around the south pole (*left*) and around a latitude of 60° southeast (*right*) in order to show latitudinal differences in the spectral range extents of $1/f$ interval (see text). *Bottom*: Lyα power spectra from LS analysis (*solid lines*) averaged over the same latitude intervals as in the top panels and the corresponding fitting functions (*dotted lines*); reference solid lines show the f^{-2}, f^{-1}, and f^0 slopes.

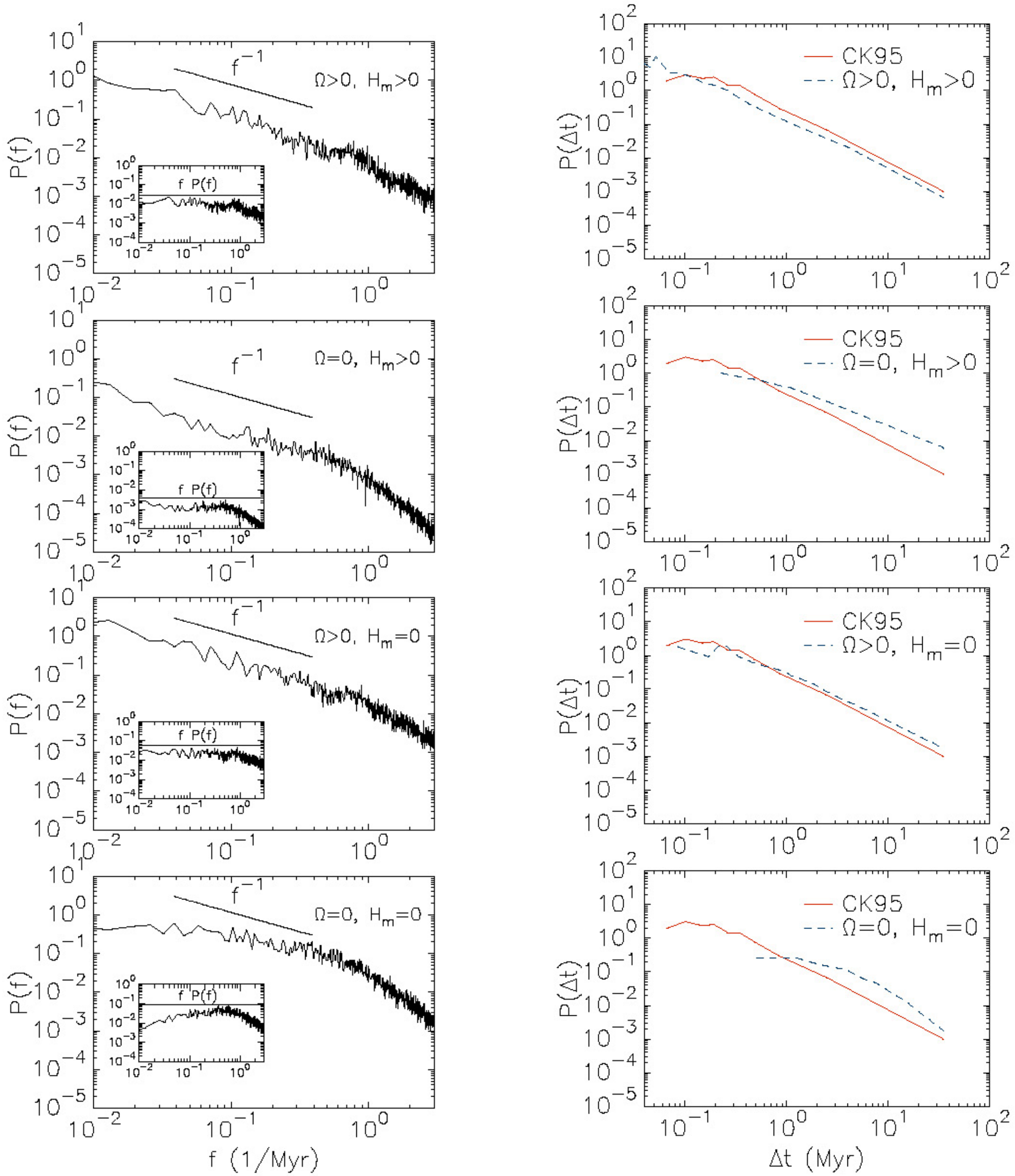
Matthaeus et al, ApJ 2007
Bemporad et al, ApJ 2008

1/f noise and reversals in spherical MHD dynamo

Incompressible MHD spherical Galerkin model low order truncation

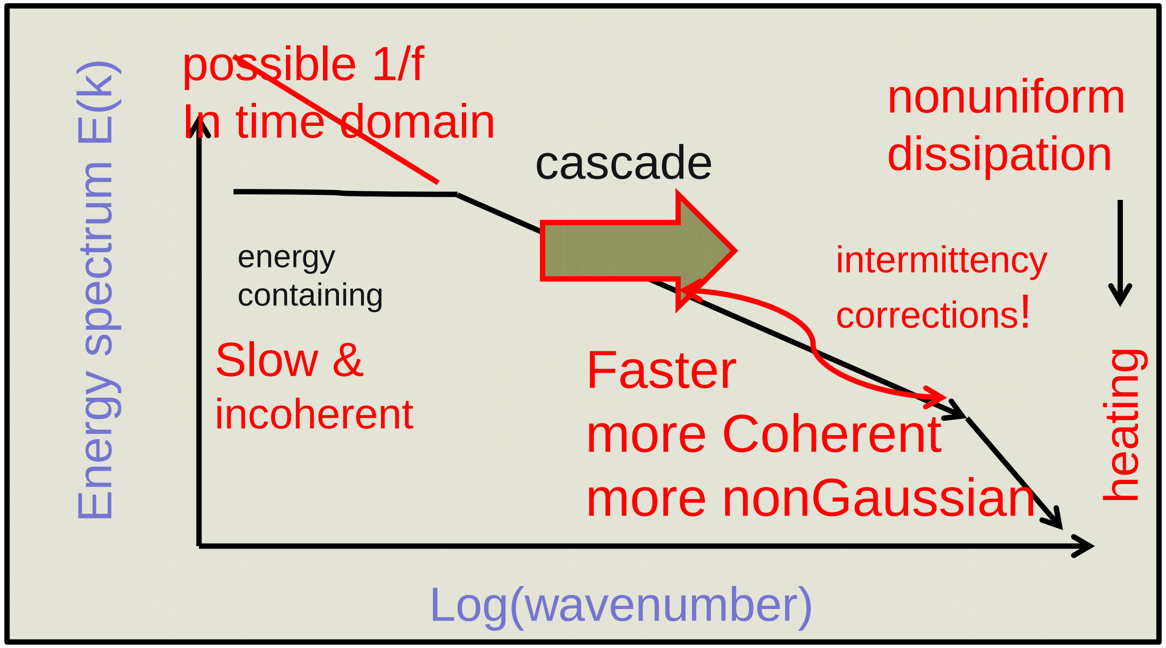
→ Run for 1000s of Tnl
→ See ramdon reversals of the dipole moment
→ 1/f noise with rotation and or magnetic helicity

Dmitruk et al, PRE in press 2014



With rotation/helicity → Waiting times for reversals scale like geophysical data!

More detailed cascade picture: central role of



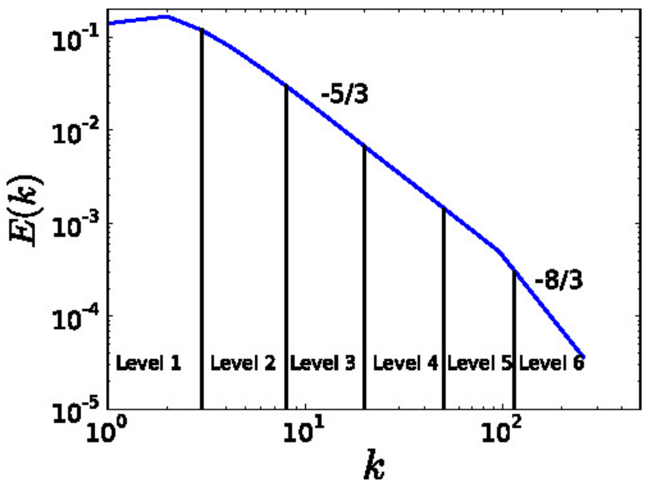
- **Cascade:** *progressively enhances nonGaussian character*
- *Generation of and patchy correlations*
- *Coherent structures are sites of*
- for inverse cascade/quasi-invariant case, **1/f noise** low frequency irregularity in time, and build up of long wavelegnth fluctuations

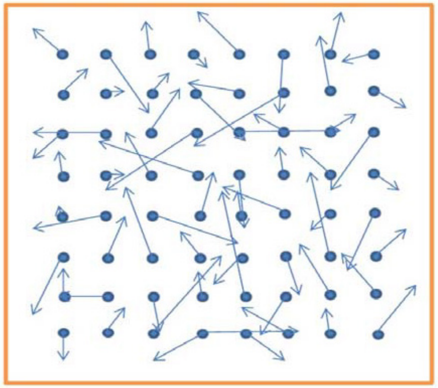
Toy model to generate intermittency

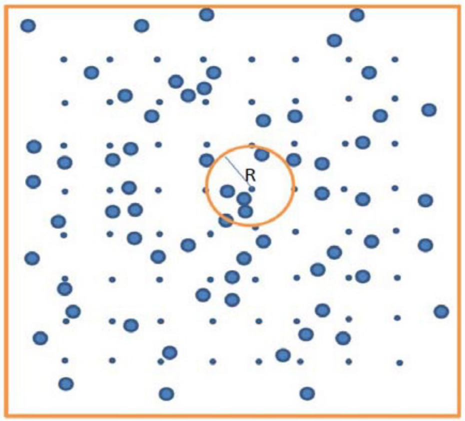
- May be useful in transport studies as an improvement over random phase data
- We already saw that structure is generated by ideal processes…so…

Synthetic realizations with intermittency

- Minimal Lagrangian Map (Rosales & Meneveau, 2006)
- Add magnetic field; map using velocity (Subedi et al, 2014)







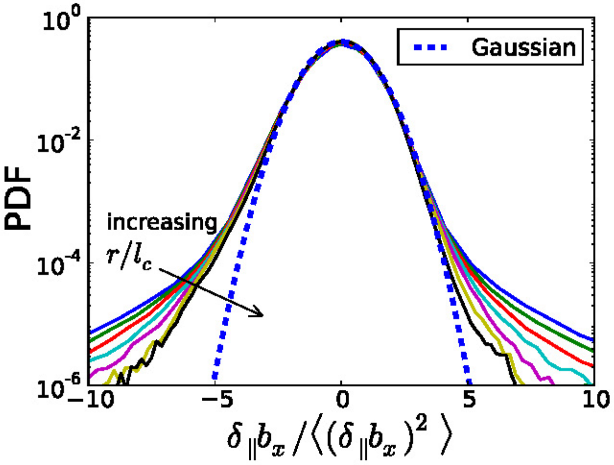
Choose spectrum
Iterate low pass filtering
get filtered fields

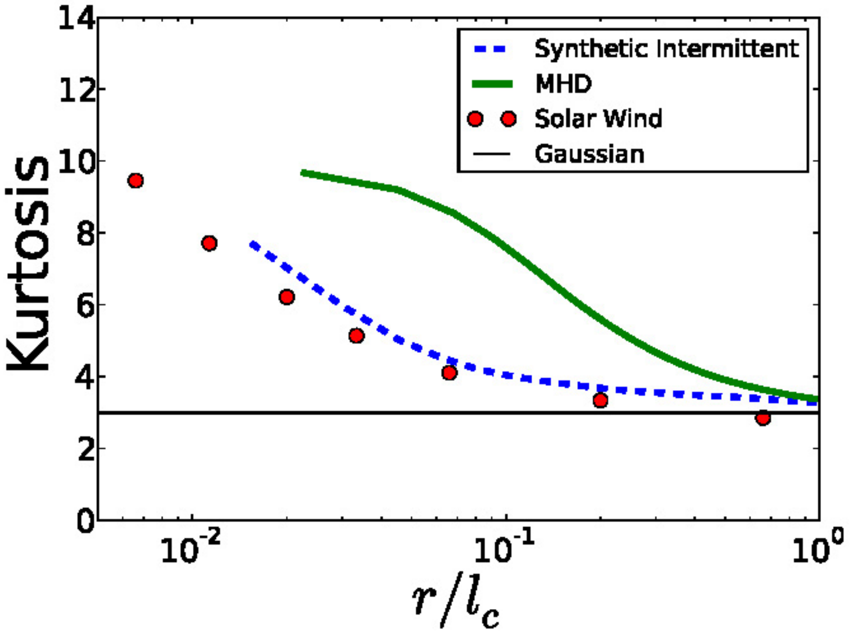
Push filtered vector fields v & b with filtered v-field at this level

Re-map onto grid by averaging; proceed to next level

After several (M=7) iterations

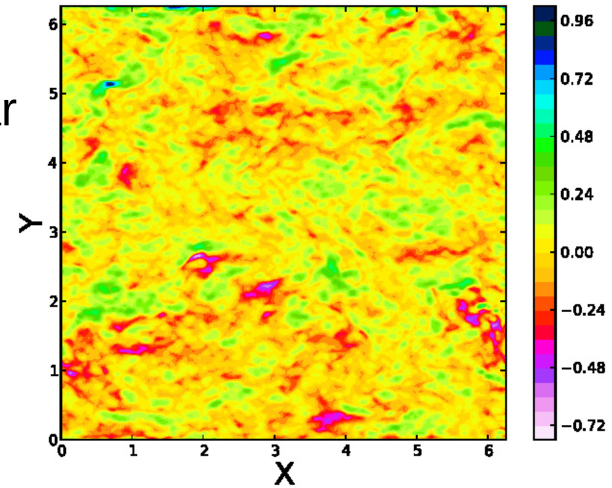
Pdfs of longitudinal magnetic increments vs lag.





Comparison of scale dependent kurtosis: SW, synthetic and MHD simulation

Perpendicular current density in a plane



summary

Intermittency is a factor in solar prediction and space weather:

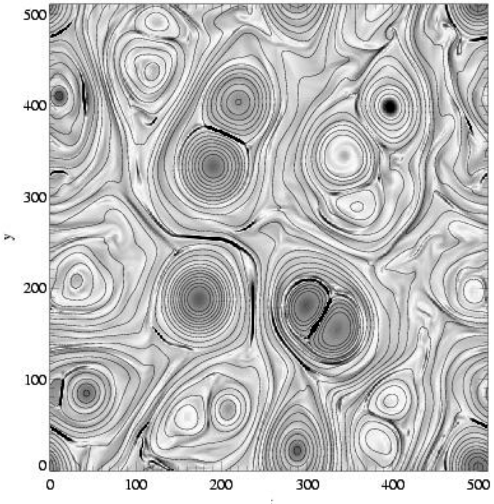
- ***Large scale/low frequency intermittency*** (1/f noise) controls
unsteady fluctuations in global parameters including extreme events

- ***Inertial range intermittency*** generates structures that channel, trap and transpsort SEPs and change connectivity of field lines

- ***Small scale (kinetic) intermittency*** implements heating and dissipation and controls reconnection rates

Coherent magnetic structures emerge in many theoretical models

Current and Magnetic field in 2D MHD simulation



3D isotropic MHD current
Mininni, NJP 2008

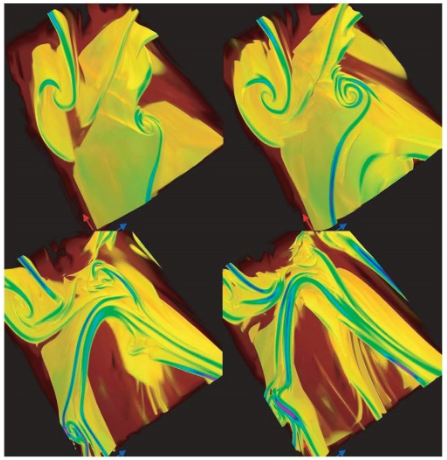
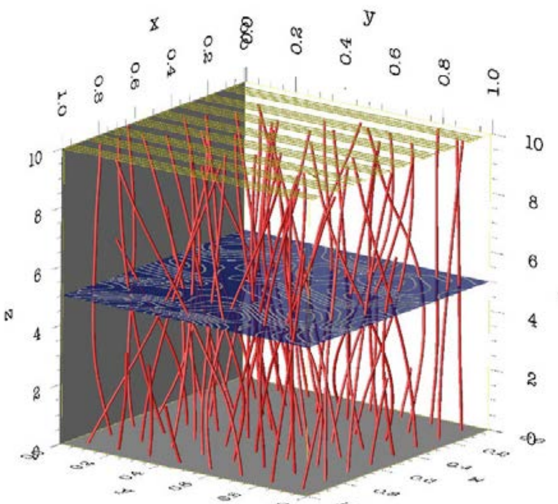
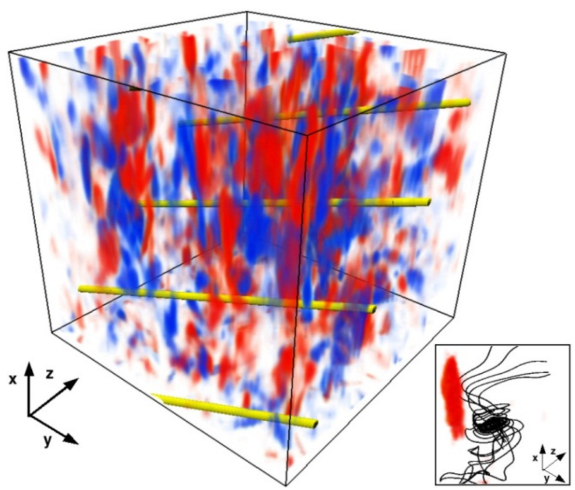


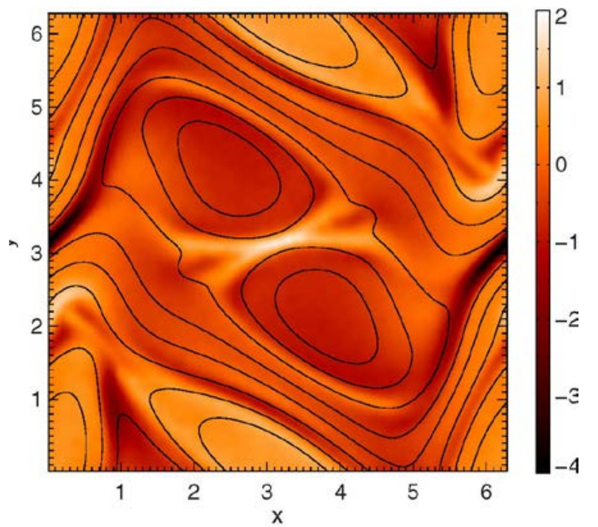
Figure 10. Same as in figure 9, but showing only the current intensity. The associated movie (available from stacks.iop.org/NJP/10/125007/mmedia) shows the temporal evolution.



Parker problem: RMHD
Rappazzo & Velli 2010



3D Hall *MHD* compressible, strong B_0, current
Dmitruk 2006



2.5D kinetic hybrid
Parashar et al, 2010